# Incomplete Exchange Rate Pass-Through, Imperfect Financial Market Integration and Optimal Monetary Policy

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**Abstract**

How do incomplete exchange rate pass-through and imperfect financial market integration affect the goal and the effectiveness of optimal monetary policy? We augment a standard monetary open economy model by incorporating both heterogeneity among exporters in their currency invoicing and capital controls, and generalize the global loss function in Corsetti et al. (2020) to the economy in which producers of producer currency pricing (PCP) and local currency pricing (LCP) coexist. Our global loss function suggests that cooperative policymakers should concern price dispersion among imported goods which are invoiced in different currencies because exchange rate fluctuations disperse their retail prices under nominal rigidities. We characterize optimal targeting rules under special parameter values and show that the policy stance is independent of the degree of cross-country risk sharing in the regime of LCP. By contrast, in the regime of PCP, the targeting rule implies that PPI inflation should respond more to external target and less to output gap as the cross-country financial integration deteriorates under the trade elasticity greater than unity. Our numerical analysis reveals that the nature of shocks, the degree of exchange rate pass-through, and the degree of financial market integration jointly determine which measure of inflation (CPI vs. PPI) should be targeted by policymakers.

**Keywords:** Currency misalignments; cross-country risk sharing; demand imbalance; exchange rate pass-through; international policy coordination; multiple invoicing currencies; optimal monetary policy; pricing-to-market.

**JEL codes:** E44; E52; E61; F41; F42.

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# 1 Introduction

Incomplete exchange rate pass-through (ERPT) and imperfect cross-country risk sharing are well-established empirical regularities in international macroeconomics. Empirical literature has shown that the pass-through of exchange rate changes into import and consumer prices is incomplete; at-the-dock and retail prices do not respond one-for-one to exchange rate fluctuations, leading to the violation of the law of one price.[1]

Table 1: Exchange Rate Pass-Through in Seven Eurozone Countries (1999q1 – 2019q4)

| | FIN | FRA | DEU | GRC | ITA | NLD | ESP |
|---|---|---|---|---|---|---|---|
| Short-Run | | | | | | | |
| IPI | 0.30 | 0.17 | 0.41 | 0.10 | 0.32 | 0.52 | 0.75 |
| | (.09) | (.09) | (.05) | (.10) | (.15) | (.06) | (.10) |
| CPIT | 0.06 | 0.01 | -.05 | 0.01 | -.08 | -.06 | -.18 |
| | (.14) | (.17) | (.04) | (.29) | (.05) | (.11) | (.33) |
| Long-Run (2 years) | | | | | | | |
| IPI | 0.41 | 0.24 | 0.65 | 0.30 | 0.53 | 0.71 | 0.59 |
| | (.18) | (.38) | (.09) | (.16) | (.26) | (.13) | (.24) |
| CPIT | 0.43 | 0.17 | 0.05 | 0.18 | 0.08 | 0.47 | 0.09 |
| | (.17) | (.20) | (.05) | (.25) | (.11) | (.17) | (.29) |

***Note*** – IPI denotes the import price index and CPIT represents the consumer price index of tradeable items. We report ERPT in seven Eurozone countries due to the IPI data availability in Eurostat and other sources. We redo pass-through regressions in Burstein and Gopinath (2014) using updated series. BIS effective exchange rate (narrow indices) is used for the nominal exchange rate. See the appendix for data description and more results from various countries. Data span 1999q1 to 2019q4 if available: IPI (1999q1-2019q4) and CPIT (1999q1-2019q4) for FIN; IPI (1999q2-2019q4) and CPIT (1999q1-2019q4) for FRA; IPI (1999q1-2019q4) and CPIT (1999q1-2019q4) for DEU; IPI (2000q2-2019q4) and CPIT (1999q1-2019q4) for GRC; IPI (1999q1-2019q4) and CPIT (1999q1-2019q4) for ITA; IPI (2000q2-2019q4) and CPIT (1999q1-2019q4) for NLD; IPI (2005q2-2019q4) and CPIT (1999q1-2019q4) for ESP.

Table 1 shows the result from pass-through regressions in seven countries in the euro area with quarterly series from 1999 to 2019.[2] The pass-through regression implies that the ERPT

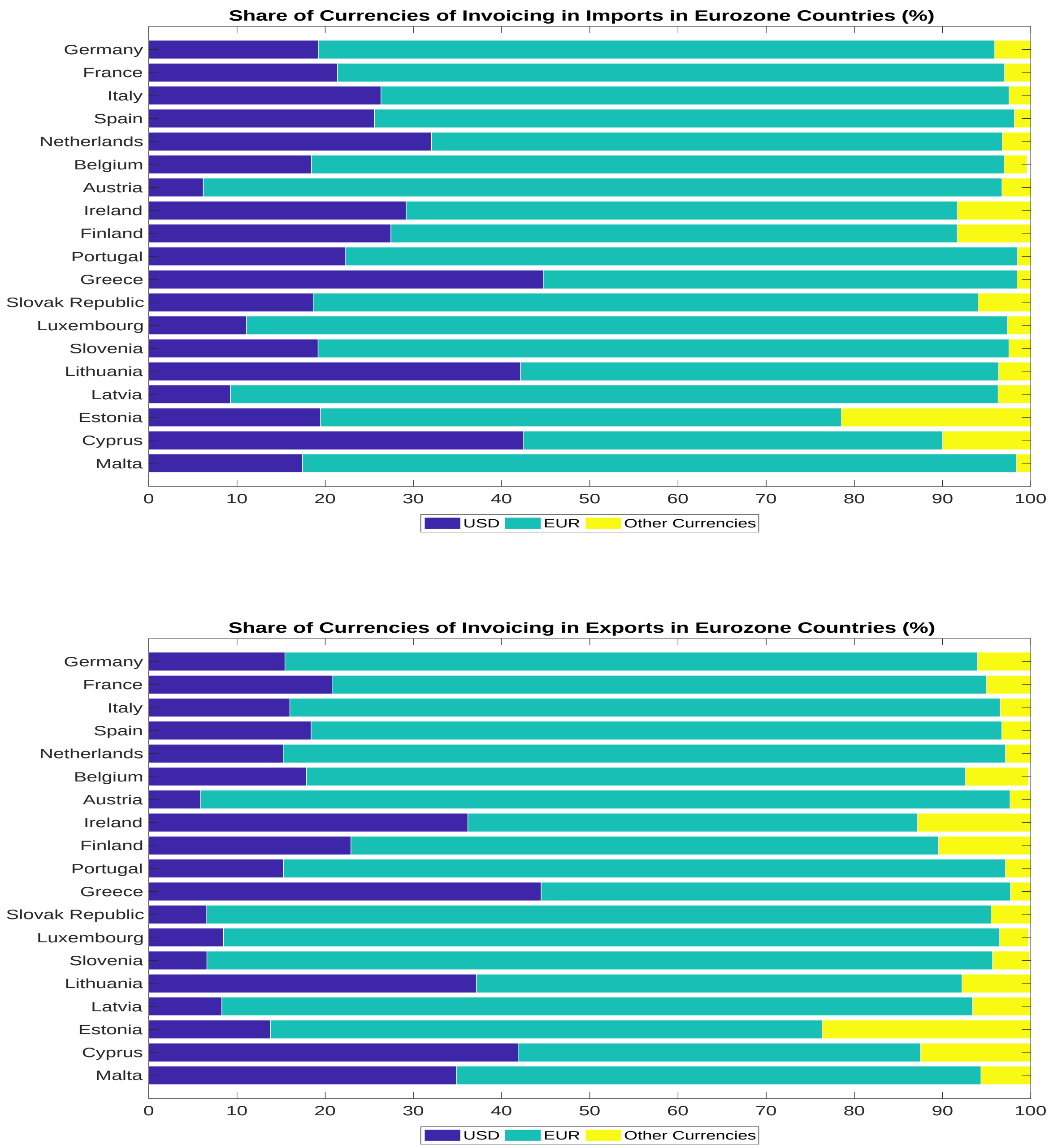
[1] The literature on incomplete exchange rate pass-through (ERPT) and pricing-to-market is comprehensively surveyed in Campa and Goldberg (2005) and Campa and Goldberg (2010) for a series of cross-country empirical studies and in Burstein and Gopinath (2014), Devereux et al. (2017), Chen et al. (2019), Corsetti et al. (2020), and Bonadio, Fischer, and Sauré (2020) for recent work using detailed goods-level pricing data.

[2] We follow pass-through regressions in Burstein and Gopinath (2014). They estimate exchange rate pass-through by using the regression equation:

$$\Delta p_{n,t} = \alpha_n + \sum_{k=0}^{8} \beta_{n,k} \Delta e_{n,t-k} + \gamma_{in} \mathbf{X}_{n,t} + \epsilon_{n,t}.$$

Here, the time frequency for $t$ is quarterly. $\Delta p_{n,t}$ denotes log changes in import price index or tradeable consumer price index in terms of country $n$'s currency. $\Delta e_{n,t}$ represents log changes in the trade-weighted nominal exchange rate. BIS effective exchange rate (narrow index) is used for the nominal exchange rate index. $\mathbf{X}_{n,t}$ represents a vector of controls: lags 0 to 8 of log changes in the trade-weighted producer price index averaged over country $n$'s trade partners. $\beta_{n,0}$ measures short-run pass-through and $\sum_{k=0}^{8} \beta_{n,k}$ estimates long-run pass-through. Standard errors are

Figure 1: Currencies of Invoicing in Eurozone Countries




***Note –*** Data are from Boz et al. (2020). Eurozone countries are listed in a declining order of Nominal GDP in 2019. See figures 13 and 14 in section K in the online appendix for currency composition of trade in European countries outside the euro area and countries outside Europe.

to both import price index (IPI) and tradeable consumer price index (CPIT) inflation rates is

calculated using the Newey-West HAC estimator with a bandwidth of 8 lags.

incomplete for seven Eurozone countries: Finland, France, Germany, Greece, Italy, Netherlands, and Spain. Among these economies in the euro area, the cumulative (long-run) pass-through from 10% exchange rate depreciations every quarter in the past two years amounts to 4.90% increase in IPI and 2.10% increase in CPI of tradeable goods on average. In addition, Gopinath (2015) and Boz et al. (2020) document that U.S. dollars and euros are dominant currencies of invoicing in the international trade. Figure 1 presents shares of invoicing currencies in nineteen Eurozone countries.[3] Across these countries, the average shares of dollars and euros amount to 24% and 71% for imports and 20% and 73% for exports, respectively. This fact indicates that there is heterogeneity among Eurozone exporters and importers in terms of their invoicing currencies and the degree of pass-through is determined by the relative fractions among different invoicing currencies.[4]

The extent to which exchange rate fluctuations affect prices is crucial to set monetary policy appropriately. If the pass-through were complete, any change in the nominal exchange rate would fully adjust prices of imported goods relative to domestically-produced goods and the law of one price would hold. This implies that the monetary authority only concerns the traditional trade-offs between inflation and output stabilization; the policymaker should let exchange rates flexibly determined at market levels and adjustments of relative prices of goods induce an efficient switch in expenditures (Friedman (1953)). However, if the law of one price fails due to the presence of nominal rigidities under pricing-to-market[5], then there is distortion in relative prices not only within a country, but also across countries in terms of common currency. This relative price distortion expands the policy trade-offs to include "external" objectives: targeting currency misalignment (see Engel (2011)).

On the other hand, empirical evidence points to the fact that risk sharing across countries is imperfect. Since the early 1980s, cross-country ownership of foreign assets has dramatically increased and international financial integration has broadened investment opportunity sets fundamentally.

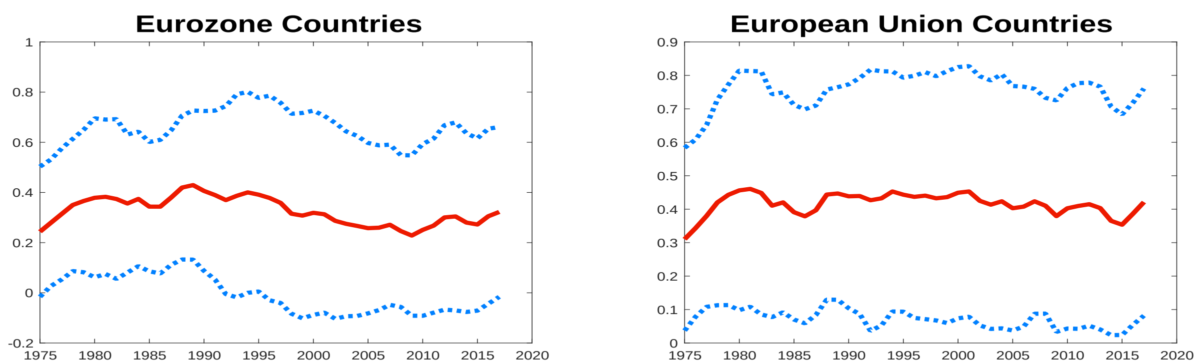
[3]Data are from Boz et al. (2020). To construct time-invariant shares of invoicing currencies in each country, we take simple averages across years from 1999 to 2019 and thus they need not sum up to 100%. We report time averages because shares of currencies are stable over time. The bar plot in Figure 1 is truncated at 100%.

[4]The degree of exchange rate pass-through is determined not only by the relative shares of invoicing currencies, but also by the duration between price adjustments. Under the standard two-country DSGE framework with full price stickiness, if some exporters set prices in the producer's currency (producer-currency pricing or PCP) and the others invoice in the consumer's currency (local-currency pricing or LCP), the degree of exchange rate pass-through varies from 0% to 100%.

[5]In general, the term 'pricing-to-market' refers to third-degree price discrimination across different export destinations. In one strand of the literature under flexible prices, this reflects the price-setting behavior in which exporters charge different markups across different markets. In the other strand of the literature under sticky prices, exporters price their products in the respective currency of the destination market and their price adjustments are sluggish in the currency of invoicing. This paper adopts the latter approach.

The conventional theory predicts that if cross-country financial integration is perfect, efficient capital flows equalize consumption growth rates across countries. When countries are more financially integrated, idiosyncratic shocks to consumption are diversified away and shared with other countries so that they are capable of better consumption smoothing. Puzzlingly, there is an extensive literature documenting that the degree of international risk sharing is at best modest and far from the levels predicted by the benchmark international business cycle model.[6]

Figure 2: Cross-Country Consumption Risk Sharing




***Note*** – We plot an estimate $\beta_t$ over time from a regression: $\Delta \log C_{it} - \Delta \log C_t = \alpha_t + \boldsymbol{\beta_t} (\Delta \log Y_{it} - \Delta \log Y_t) + \epsilon_{it}$ by averaging over a 15-year rolling window. The vertical axis represents the degree of risk sharing in each country group, ranging from 0 (perfect risk sharing) to 1 (no risk sharing). $\Delta$ denotes a 1-year difference operator. See Figure 15 for results from more country groups and see figures 16 and 17 for results from 5 and 10 year differences in the online appendix K. The data source is Penn World Tables 9.1.

Figure 2 presents empirical results from a risk-sharing regression analysis.[7] Each chart displays the degree of cross-country risk sharing ranging from zero (perfect risk sharing) to one (no risk sharing) over time by country group: Eurozone countries and European Union. It is apparent that in spite of much progress in financial globalization over the last four decades, the regression estimate has been far from zero for European country groups and it indicates that cross-country consumption

[6]See Backus et al. (1992), Lewis (1996), Chari et al. (2002), Kose et al. (2009), and Bai and Zhang (2012) among others.

[7]To be specific, we estimate a 15-year rolling window average $\beta_t$ from a regression equation given by

$$\Delta \log C_{it} - \Delta \log C_t = \alpha_t + \boldsymbol{\beta_t} (\Delta \log Y_{it} - \Delta \log Y_t) + \epsilon_{it} \quad \text{for country } i \text{ and time } t,$$

where given country $i$, $C_{it}$ is per-capita real consumption of households and government at current PPPs (2011 USD) and $Y_{it}$ is per-capita real GDP at current PPPs (2011 USD). We repeat the above regression by country group: Eurozone and European Union. $C_t$ and $Y_t$ denote total consumption and output from all countries in each group: $C_t = \sum_i C_{it}$ and $Y_t = \sum_i Y_{it}$ for $i \in$ a group of countries. Perfect cross-country risk sharing implies that the country-specific consumption growth rate relative to its country-group counterpart is uncorrelated with its relative output growth rate, so that the coefficient $\beta_t$ should be zero across time. $\beta_t$ ranges from 0 (perfect risk sharing) to 1 (no risk sharing) and captures the average degree of synchronization between relative consumption and output growth rates. By subtracting $C_t$ ($Y_t$) from $C_{it}$ ($Y_{it}$), we eliminate globally common factors in regression variables.

risk sharing has been limited. For the policy implication, imperfect financial market integration induces inefficient movements in cross-border capital flows, which in turn leads to deviations from perfect risk sharing. This financial market imperfection distorts current account positions across countries and adds another dimension to policymakers' trade-offs: targeting cross-country demand imbalance (see Corsetti et al. (2010) and Corsetti et al. (2020)).

Considering these stylized facts, we augment a standard New Keynesian open economy model by allowing for incomplete exchange rate pass-through and imperfect financial market integration, and characterize optimal monetary policy. The crucial departure from the literature on optimal monetary policy in micro-founded New Keynesian open economy models is twofold. First, we introduce heterogeneity among exporters in terms of invoicing currencies under nominal rigidities. A fraction of exporters set their prices in the producer's currency (producer currency pricing or PCP) as in the classic Mundell-Fleming paradigm while the remainder of producers segment markets by country and set prices in the importer's currency (local currency pricing or LCP) as in the Betts-Devereux-Engel paradigm.[8] By varying the relative share of imported goods priced in the producer's currency versus local currency, the degree of short-run ERPT ranges from 0% to 100% if prices are fully sticky in the invoicing currency. Second, our model captures variation in the degree of financial market integration ranging from perfect cross-country risk sharing to financial autarky through capital controls. In the model, there are state-contingent claims for all states of nature but security returns are distorted by state-contingent wedges (see Devereux and Yetman (2014a) and Devereux and Yetman (2014b)).[9] Many countries introduced controls on capital flows for several rationales which involve countervailing currency appreciation, regulating sudden injection or withdrawal of funds (hot money), and monetary policy autonomy. We explore the effect that the introduction of capital controls has on optimal monetary policy when the degree of short-run ERPT changes.

By constructing the theoretical laboratory featuring incomplete ERPT and imperfect risk sharing, this paper aims at addressing the question: how do the degrees of ERPT and imperfect financial integration shape the objective and the effectiveness of optimal monetary policy under cooperation? In a two-country open economy model, we generalize the quadratic loss function derived in

[8] See Betts and Devereux (2000), Devereux and Engel (2003), Engel (2011) and others.

[9] Trade and financial flows can be hampered due to capital controls in reality. The price control takes the form of taxes on returns to international investment or taxes on certain types of transactions. The quantity control imposes quotas or outright prohibitions on asset holdings and loan portfolio. See Magud et al. (2018) for more details. Our model captures capital controls by imposing taxes on cross-country financial capital flows as in Devereux and Yetman (2014a) and Devereux and Yetman (2014b).

Corsetti et al. (2020) and analytically show policymakers should concern relative price differences originating from heterogeneity among exporters in their invoicing currencies. We find that under perfect financial markets, the coexistence of PCP and LCP goods in export markets creates new quadratic loss terms in cooperative central banks' objective function because exchange rate movements disperse consumer prices of PCP and LCP exported goods. This new channel is absent in the two polar regimes of PCP and LCP.

Suppose cross-country risk sharing is perfect. Under PCP, the law of one price holds. If domestic price inflation (PPI or GDP deflator inflation) is fully stabilized, there is no price dispersion in both domestic and export markets. On the other hand, when export prices are sticky in the importer's currency (LCP), identical goods are sold at different prices across countries in common currency, leading to resource (labor) misallocation. In addition, since only firms who are allowed to reoptimize reflect the effect of exchange rate movements on their marginal revenues, exchange rate fluctuations exacerbate price dispersion between two groups of LCP products: goods of no price change and goods newly priced. Among LCP goods of no price change, exchange rate changes do not incur further price distortion.

If we extend the model to incorporate heterogeneity among exporters in terms of invoicing currencies, exchange rate movements create additional resource misallocation other than what we describe above. When prices are fully rigid in invoicing currencies, consumer prices of PCP exported goods fluctuate in accordance with exchange rate changes while consumer prices of LCP exported goods stay fixed, causing cross-sectional price dispersion even among goods whose prices are not adjusted by their producers. Moreover, note that price dispersion is persistent. Contrary to prices of LCP goods under nominal rigidity, destination-currency prices of PCP exported goods respond one-for-one to exchange rate movements every period and this fact cumulatively impinges on resource allocation between PCP and LCP firms in a dynamic setting.

After deriving a global loss function under generic degrees of pricing-to-market and risk sharing, we restrict our focus on the two polar regimes of PCP and LCP, and characterize simple targeting rules to investigate the effect of imperfect financial integration. Following the literature, we consider only the special case in which preferences are log in consumption and linear in leisure.[10] It turns out that under LCP, the degree of financial integration does not affect the targeting rule which dictates how aggressively the central bank should adjust relative inflation in response to relative

[10]See Devereux and Engel (2003), Corsetti and Pesenti (2005), Corsetti et al. (2010), Engel (2011) and Corsetti et al. (2020) among others.

output gap and external imbalance (currency misalignment plus cross-country demand imbalance). Regardless of the degree of cross-country risk sharing, the policy stance is "leaning against the wind": the policymaker curbs home CPI inflation in response to the increase in relative output gap and/or external imbalance, holding foreign CPI inflation fixed.

By contrast, under PCP, we find that monetary policy management depends on the degree of financial integration and its qualitative implications change according to the substitutability between home and foreign goods.[11] If the trade elasticity is greater than unity, the implication from the targeting rule is the same as "leaning against the wind." Holding foreign PPI inflation constant, the global planner contracts home PPI inflation in accordance with the rise in relative output gap and/or demand imbalance. The worse degree of cross-country financial integration makes the inflation responsiveness higher to external target (i.e. demand imbalance) and lower to internal target (i.e. output gap). Conversely, if the trade elasticity is lower than unity and cross-country risk sharing deteriorates, the policy prescription suggests relative PPI inflation should respond more aggressively to both relative output gap and demand imbalance but these responses are in the opposite direction. That is, relative PPI inflation should fall in response to the rise in relative output gap while the policymaker raises inflation according to the increase in demand imbalance.

Lastly, we draw out implications on strict inflation targeting under calibrated parameter values which do not allow for analytical targeting rules in closed-form. Since the numerical analysis of optimal monetary policy lacks operational guidance for the policymaker on the implementation of monetary policy, we provide the comparison between optimal monetary policy and strict inflation targeting. Under our benchmark parametrization where the trade elasticity is greater than unity, we show that the nature of shocks, the degree of ERPT and the degree of financial market integration determine which measure of inflation should be targeted by cooperative policymakers. We find that in response to supply shocks, the central bank should target PPI (CPI) inflation in the high (low) ERPT regime regardless of the degree of financial market integration. On the other hand, if shocks originate from the demand side, CPI inflation targeting is valid only under the low ERPT regime and the high degree of financial market integration. If the cross-country risk sharing is close to the level under financial autarky, PPI inflation targeting achieves a level of welfare closer to optimal monetary policy regardless of the degree of ERPT in response to preference shocks. The reason is the following. If the economy is close to financial autarky and the degree of ERPT is

[11] Following the literature, we refer to the substitutability between home and foreign goods as the trade elasticity.

low, volatility from external targets incurs larger welfare loss than that from internal targets.[12] It turns out that CPI inflation stabilizes output gap excessively by trading off higher volatility from external targets whereas PPI inflation targeting stabilizes external targets to the similar extent as optimal monetary policy does. This drives lower welfare loss under strict PPI inflation targeting than CPI inflation targeting under preference shocks.

**Contribution to the Literature:** Characterizing optimal monetary policy under a joint theoretical framework which features multiple invoicing currencies and imperfect financial market integration is novel in the literature and it is an important contribution of this paper. To differentiate this paper from other studies, it is helpful to place our work relative to two strands of papers regarding incomplete ERPT and imperfect financial markets.

First, there are three leading explanations for incomplete ERPT. The first channel is short-run nominal rigidities combined with local currency pricing (Gopinath and Rigobon (2008), Gopinath et al. (2010), Gopinath and Itskhoki (2010)). Another possibility is that exporting firms differentially adjust their markups across destinations to accommodate the local market environment (Krugman (1986), Dornbusch (1987), Atkeson and Burstein (2008)). The other rationale is the presence of local distribution costs in the destination country (Burstein et al. (2003), Corsetti and Dedola (2005)). Our analysis builds on the first approach and abstracts from strategic complementarities in price setting or distribution costs in order to draw out the clear implications for monetary policy. Clarida et al. (2002) and Benigno and Benigno (2006) develop a two-country model which assumes perfect ERPT and delve into the analysis of cooperative and non-cooperative optimal monetary policies. Engel (2011) introduces LCP into the model of Clarida et al. (2002). Engel (2011) shows that inward-looking policies[13] are inefficient and the policymaker must target exchange rate misalignment when ERPT is incomplete. Fujiwara and Wang (2017) revisit the desirability of monetary policy cooperation and compare equilibrium outcomes under cooperative and non-cooperative policies in a low pass-through environment (LCP).

Using staggered price setting á la Calvo (1983)[14], these papers discuss implications on policy trade-offs in an open-economy model with and without commitment (Engel (2011)) or with and without cooperation (Clarida et al. (2002), Benigno and Benigno (2006), Fujiwara and Wang

[12] By external targets, we mean currency misalignment and cross-country demand imbalance. By contrast, internal targets consist of inflation and output gap.

[13] By a "inward-looking" policy, we mean the goal of that policy is only twofold as in the standard closed economy: stabilizing inflation and closing output gap.

[14] For important contributions with one-period ahead price setting, see Corsetti and Pesenti (2001), Benigno and Benigno (2003), Devereux and Engel (2003) and Corsetti and Pesenti (2005).

(2017)).[15] Since all these papers assume single-currency invoicing among exporters, the degree of short-run ERPT is fixed at some certain level close to 0% (LCP) or 100% (PCP) under sticky prices in the invoicing currency. Moreover, they assume asset markets are frictionless. If cross-country risk sharing is perfect, currency misalignment is the only external target for policymakers under incomplete ERPT. This paper extends their analysis to a more flexible framework in which the degree of ERPT ranges from full LCP to full PCP and the degree of financial integration varies from financial autarky to perfect risk sharing. We analytically show that the coexistence of PCP and LCP firms causes additional relative price distortions in export markets which should be another concern for cooperative policymakers.

Second, our paper is related to a recent body of literature on optimal monetary policy in imperfect international financial markets. Corsetti et al. (2010), Engel (2016) and Corsetti et al. (2020) extend Engel (2011) and analyze optimal monetary policy when international financial trade is absent or confined to an uncontingent bond. Most closely related to our work is the study by Corsetti et al. (2020). Using the workhorse two-country New Keynesian model where the only asset is an uncontingent bond, they show there exists another external policy target, i.e., cross-country demand imbalance in addition to currency misalignment if financial markets are imperfect. Under the Cole and Obstfeld (1991) specification, they find that the optimal policy stance is contractionary in the low ERPT (LCP), expansionary in the high ERPT (PCP), in response to inefficient capital inflows. As compared to the contribution of Corsetti et al. (2020), this paper establishes the equivalence result of optimal policy responses between our economy and the economy with uncontingent bonds when capital flows are exogenous to policy. Under the Cole and Obstfeld (1991) specification, we show that net exports and capital flows in our model are not affected by policy as in the economy with uncontingent bonds. Therefore, all qualitative policy responses discussed in Corsetti et al. (2020) carry over to our economy.[16] The main departure of this paper from their work is to incorporate multiple-currency invoicing and capital controls under complete asset markets. We generalize their loss function to the framework which nests PCP and LCP. Our numerical analysis reveals that under incomplete ERPT, capital controls can improve the global welfare but it turns out that the welfare gains are very small. Importantly, we discuss how the

[15] Engel (2011) focuses on the case under international cooperation. Clarida et al. (2002) assume the monetary authorities lack a commitment technology and examine the analogies of policy in open and closed economies. Central banks in Benigno and Benigno (2006) and Fujiwara and Wang (2017) conduct optimal commitment policy from the timeless perspective as in Woodford (2003).

[16] This holds true only under the Cole and Obstfeld (1991) specification. If we deviate from the unitary trade elasticity, then policy responses in our economy differ from those in Corsetti et al. (2020).

degree of international risk sharing analytically shapes optimal targeting rules under PCP, which is absent in Corsetti et al. (2020).

Modeling financial market imperfections through a state-contingent wedge in security returns was developed in Devereux and Yetman (2014a) and Devereux and Yetman (2014b). Our financial market specification is grounded in their theoretical framework. Devereux and Yetman (2014b) studies monetary policy under incomplete ERPT and imperfect risk sharing with a focus on sterilized intervention. They, however, assume monetary policy is governed by an ad-hoc instrument rule. Under the similar framework for financial markets, Devereux and Yetman (2014a) studies optimal monetary policy in the presence of capital controls and the zero-bound constraint on interest rates. They assume PCP and investigate the issue of policy trilemma under a liquidity trap. Engel (2014) synthesizes literature on optimal monetary policy under pricing-to-market and incomplete financial markets. But these papers are not clear on whether there exist separate policy objectives arising from multiple-currency invoicing of exporters. The main contribution of this paper is to show exchange rate movements disperse consumer prices of PCP and LCP products in export markets, leading to an additional loss term in the cooperative policymaker's external objective. In addition, the above papers do not discuss the effect of imperfect risk sharing on optimal targeting rules under LCP and PCP. Our paper fills this gap.[17] Our paper also discusses which inflation measure (CPI vs. PPI) should be stabilized according to the nature of shocks (supply or demand), the degree of ERPT and the extent of cross-country risk sharing in providing guidance on inflation targeting.

The remainder of the paper is organized as follows. Section 2 specifies the model and presents equilibrium conditions. Section 3 explains log-linearized equilibrium conditions. Section 4 characterizes optimal monetary policy and discusses policymakers' objectives and targeting rules. Section 5 assesses numerical results and concluding remarks are given in section 6. We relegate technical derivations and the model details to the online appendix.[18]

[17]Under the same framework, Engel (2014) shows that optimal targeting rules under LCP do not depend on the degree of cross-country risk sharing. The effect of imperfect risk sharing on optimal targeting rules under PCP is newly discussed in this paper.

[18]The online appendix can be found in the author's webpage (http://econhanwt.github.io/).

## 2 The Model

The baseline framework closely follows Engel (2011) and Corsetti et al. (2020). We extend Engel (2011) in four dimensions. First, we deviate from the unitary elasticity of substitution between domestic and imported consumption indexes as in Corsetti et al. (2020). Second, international financial markets are imperfect. A complete set of contingent money claims is traded across countries, but there is a wedge in the returns to contingent claims for households. By this wedge, we have varying degrees of international financial integration ranging from financial autarky to perfect cross-country risk sharing (Devereux and Yetman (2014a) and Devereux and Yetman (2014b)). Third, we allow for generic degree of exchange rate pass-through. In our model, short-run exchange rate pass-through into import prices ranges from 0% to 100% as in Betts and Devereux (2000) under full price rigidities.[19] Lastly, the economy is perturbed by not only supply shocks to wage markups and productivity, but also demand shocks to preference. We analyze the effects of demand and supply shocks under optimal monetary policy.[20]

Figure 3 displays the structure of the model. There are two countries labeled by "Home" and "Foreign." We think of Eurozone countries as the home country while the foreign country corresponds to the rest of the world. For simplicity, Home and Foreign are symmetric. A continuum of households of unit mass resides in each country and households obtain utility from all goods produced in both countries. Labor markets are monopolistically competitive. Each household offers a differentiated type of labor to firms located within its country. Each differentiated labor is indexed by $h$ ($h^*$) $\in [0, 1]$ for a Home (Foreign) household.

There is a continuum of differentiated tradeable goods that each country specializes in. A monopolistic firm produces each brand indexed by $f$ ($f^*$) $\in [0, 1]$ in Home (Foreign). Firms produce output using only labor. A fraction $\chi$ of firms sets export prices in the currency of the destination country (LCP) and the remaining fraction of firms, given by $1-\chi$, prices in the producer's currency (PCP). By changing the parameter $\chi$ from zero to one, our analysis subsumes both PCP and LCP.

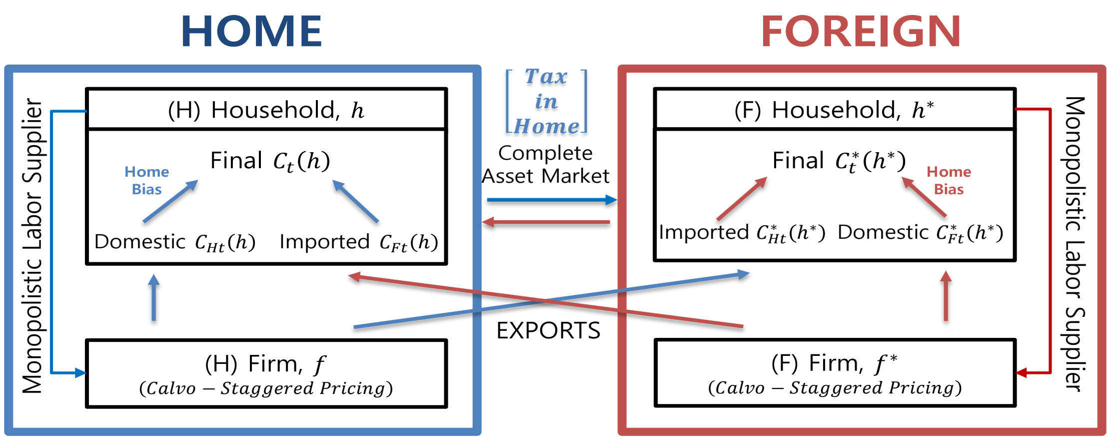
Households in Home and Foreign trade a complete set of contingent claims in international

[19] The exchange rate pass-through is 0% only if all firms who price in the consumer's currency are unable to reset their prices. Marginal revenues of LCP firms change in accordance with nominal exchange rate fluctuations and Calvo staggered pricing implies that a certain fraction of firms adjust prices every period. Hence the short-run exchange rate pass-through is above zero under LCP sticky prices á la Calvo (1983) and its degree depends on the duration of price changes.

[20] The presence of preference shocks complicates the quadratic representation of the periodic utility as shown in Corsetti et al. (2020). We extend their quadratic loss function to generic exchange rate pass-through ranging from full LCP to full PCP. See the online appendix G.

financial markets. To capture the empirical fact on imperfect risk sharing, we introduce a wedge in the security returns as in Devereux and Yetman (2014a) and Devereux and Yetman (2014b). Since the model is almost identical to Engel (2014)'s, we defer most of the details to an online appendix. In what follows, we will focus on Home agents' problems. Foreign agents' problems can be specified analogously. Foreign counterparts to Home variables are marked with an asterisk.

Figure 3: Model Structure




| | **(H) Household, $h$** | **(H) Firm, $f$: PCP$(1-\chi)$ and LCP$(\chi)$** |
|---|---|---|
| **PCP** | $\int_\chi^1 [P_{Ht}(f) C_{Ht}(h,f)]df + \int_\chi^1 [\mathcal{E}_t P^*_{Ft}(f^*) C_{Ft}(h,f^*)]df^*$ | $P_{Ht}(f)\int_0^1 [C_{Ht}(h,f)]dh + P_{Ht}(f)\int_0^1 [C^*_{Ht}(h^*,f)]dh^*$ |
| **LCP** | $\int_0^\chi [P_{Ht}(f) C_{Ht}(h,f)]df + \int_0^\chi [P_{Ft}(f^*) C_{Ft}(h,f^*)]df^*$ | $P_{Ht}(f)\int_0^1 [C_{Ht}(h,f)]dh + \mathcal{E}_t P^*_{Ht}(f)\int_0^1 [C^*_{Ht}(h^*,f)]dh^*$ |

## 2.1 Households

The representative household in the Home country maximizes

$$U_t(h) \equiv \mathbb{E}_t \sum_{j=0}^{\infty} \left[ \beta^j \left\{ \zeta_{C,t+j} \frac{[C_{t+j}(h)]^{1-\sigma}}{1-\sigma} - \kappa \frac{[N_{t+j}(h)]^{1+\phi}}{1+\phi} \right\} \right] \text{ with } \sigma > 0 \text{ and } \phi \geq 0, \tag{1}$$

where $\zeta_{C,t}$ represents a shock to preferences (a "demand" shock) and hence households increase their valuation of today's consumption relative to future consumption in response to a positive $\zeta_C$ shock. $\sigma$ denotes the inverse of the intertemporal elasticity of substitution and $\phi$ is the inverse of the Frisch labor-supply elasticity. $\kappa$ adjusts the weight given to disutility from labor. $N_t(h) \equiv \int_0^\chi N_t^L(h,f)df + \int_\chi^1 N_t^P(h,f)df$ denotes an aggregate of the labor services that a Home household $h$

provides for each of a continuum of Home firms. Here we use the superscript $L$ for LCP firms and $P$ for PCP firms. $C_t(h)$ is an Armington aggregator of Home and Foreign consumption baskets, defined by

$$C_t(h) = \left[ \left(\frac{\nu}{2}\right)^{\frac{1}{\epsilon}} \left[C_{Ht}(h)\right]^{\frac{\epsilon-1}{\epsilon}} + \left(1-\frac{\nu}{2}\right)^{\frac{1}{\epsilon}} \left[C_{Ft}(h)\right]^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \text{ with } \epsilon > 0 \text{ and } \nu \in [0,2], \tag{2}$$

where the consumption baskets of Home and Foreign brands are specified as

$$\begin{aligned} C_{Ht}(h) &= \left[\int_0^{\chi} C_{Ht}^{L}(h,f)^{\frac{\xi-1}{\xi}} df + \int_{\chi}^{1} C_{Ht}^{P}(h,f)^{\frac{\xi-1}{\xi}} df\right]^{\frac{\xi}{\xi-1}} \\ C_{Ft}(h) &= \left[\int_0^{\chi} C_{Ft}^{L}(h,f^*)^{\frac{\xi-1}{\xi}} df^* + \int_{\chi}^{1} C_{Ft}^{P}(h,f^*)^{\frac{\xi-1}{\xi}} df^*\right]^{\frac{\xi}{\xi-1}} \end{aligned} \quad \text{with } \xi > 1. \tag{3}$$

Each household puts a weight of $\left(\frac{\nu}{2}\right)$ on domestic goods and $\left(1-\frac{\nu}{2}\right)$ on imported goods. The parameter $\nu$ represents the degree of goods market integration.[21] $\epsilon$ stands for the *cross-country* trade substitutability, that is, the intratemporal elasticity of substitution between Home and Foreign consumption baskets. $\xi$ indicates the elasticity of substitution among Home (Foreign) varieties. A Home household $h$'s flow budget constraint is given by

$$\begin{aligned} & P_t C_t(h) + (1+\varrho_t)\left[\sum_{\nabla_{t+1}\in\Omega} Z\left(\nabla_{t+1}|\nabla_t\right) D\left(h,\nabla_{t+1}\right) - D\left(h,\nabla_t\right)\right] + B_{t+1}(h) \\ = \; & W_t(h)N_t(h) + \left(1+i_{t-1}\right) B_t(h) + \Gamma_t + T_t. \end{aligned}$$

The household $h$ supplies its differentiated labor service $N_t(h)$ and sets the wage rate $W_t(h)$. $Z\left(\nabla_{t+1}|\nabla_t\right)$ is the price of the claim which pays one unit of Home currency contingent upon the realization of the state $\nabla_{t+1}$ at time $t+1$, conditional on the state $\nabla_t$ at time $t$. $\Omega$ represents the state space. $D\left(h,\nabla_t\right)$ denotes the nominal balance of state-contingent bonds at $t$ and $B_t(h)$ is the nominal balance of domestic uncontingent bonds at the beginning of time $t$ with the interest rate $i_{t-1}$. Households earn a share of aggregate profit $(\Gamma_t)$ from firms and receive lump-sum transfers $(T_t)$ in each period. Note that $\sum_{\nabla_{t+1}\in\Omega} Z\left(\nabla_{t+1}|\nabla_t\right) D\left(h,\nabla_{t+1}\right) - D\left(h,\nabla_t\right)$ represents net position of external bond holdings at time $t$, which corresponds to net capital outflow.

International financial markets are imperfect due to the presence of tax on net capital flows in

[21] Increases in the degree of goods market integration are captured by a reduction in the degree of consumption home bias under the restriction, $\nu \geq 1$. The presence of home bias in consumption $(\nu > 1)$ causes the equilibrium to deviate from purchasing power parity, even with the law of one price held at the level of each individual good. Hence, consumption home bias induces endogenous real exchange rate fluctuations.

Home, $\varrho_t$[22], given by

$$(1+\varrho_t)^{1-\lambda} \equiv \left(\frac{P_{Ht}C_{Ht}+\mathcal{E}_t P^*_{Ht}C^*_{Ht}}{P_{Ht}C_{Ht}+P_{Ft}C_{Ft}}\right)^{\lambda} = \left(\frac{P_{Ht}C_{Ht}+\mathcal{E}_t P^*_{Ht}C^*_{Ht}}{P_t C_t}\right)^{\lambda} \text{ with } \lambda \in [0,1], \tag{4}$$

where $\mathcal{E}_t$ denotes the nominal exchange rate (the price of Foreign currency in units of Home currency). Observe that if the parameter $\lambda$ is set to zero, the wedge $\varrho_t$ disappears, implying full international risk sharing.[23] By contrast, if $\lambda$ is set to one, trade is forced to be balanced in every period, $P_{Ft}C_{Ft} = \mathcal{E}_t P^*_{Ht}C^*_{Ht}$, resulting in financial autarky. Through the balance of payments, the value of net exports is related to net capital outflow and hence we can rewrite (4) as

$$\begin{aligned} \log(1+\varrho_t) &\equiv \left(\tfrac{\lambda}{1-\lambda}\right)\log\left(\tfrac{P_{Ht}C_{Ht}+\mathcal{E}_t P^*_{Ht}C^*_{Ht}}{P_{Ht}C_{Ht}+P_{Ft}C_{Ft}}\right) = \omega \log\left(1+\frac{\sum\limits_{\nabla_{t+1}\in\Omega} Z(\nabla_{t+1}|\nabla_t)D(h,\nabla_{t+1}) - D(h,\nabla_t)}{P_t C_t}\right), \\ &\approx \omega\left(\frac{\sum\limits_{\nabla_{t+1}\in\Omega} Z(\nabla_{t+1}|\nabla_t)D(h,\nabla_{t+1}) - D(h,\nabla_t)}{P_t C_t}\right), \end{aligned} \tag{5}$$

where $\omega \equiv \left(\frac{\lambda}{1-\lambda}\right)$ given $\lambda \in [0,1)$. $\omega$ represents the sensitivity of the tax rate with respect to the ratio of net capital outflow to consumption expenditure. From (5) it is apparent that the Home government raises the tax rate when the ratio of net capital outflow to consumption expenditure increases.[24] Larger $\lambda$ implies capital controls respond more aggressively to the change of that ratio. In the limiting case where $\lambda$ approaches one (or $\omega \to \infty$), Home households do not trade any external bonds and all financial capital flows are shut off.

[22] Given that we characterize optimal cooperative monetary policy, capital controls can be imposed on either of two countries. In the model, capital controls are implemented in Home. Foreign households are assumed to pay no tax/subsidy on external bond holdings.

[23] In order to achieve efficient risk sharing under $\lambda = 0$, we need two assumptions about the world economy at the initial time 0: ($i$) bonds are in zero net supply and ($ii$) the economy resides in the symmetrically efficient steady state. The trade is balanced at time 0 due to the assumption ($i$). Together symmetric steady-state consumption ($C_0 = C_0^*$) and the balanced-trade condition imply that purchasing power parity holds at time 0 even under the presence of consumption home bias. As a result, efficient risk sharing follows from combining first-order conditions of external bond holdings and the PPP condition.

[24] If net capital outflow is positive by $\sum\limits_{\nabla_{t+1}\in\Omega} Z(\nabla_{t+1}|\nabla_t)\, D(h,\nabla_{t+1}) - D(h,\nabla_t) > 0$, then $\varrho_t$ is positive and this corresponds to the tax on net capital outflow. On the other hand, if net capital inflow is positive by $D(h,\nabla_t) - \sum\limits_{\nabla_{t+1}\in\Omega} Z(\nabla_{t+1}|\nabla_t)\, D(h,\nabla_{t+1}) > 0$, then $\varrho_t$ is negative and this corresponds to the tax on net capital inflow. In either of the two cases, capital controls restrict capital flows across countries.

## 2.2 Firms

A monopolistic LCP (PCP) firm $f$ produces its unique type of a tradeable good according to a linear technology:

$$Y_t^L(f) = A_t N_t^L(f), \qquad Y_t^P(f) = A_t N_t^P(f). \tag{6}$$

Here superscripts $L$ and $P$ are used for LCP and PCP firms, respectively. A CES composite of differentiated Home labor services, $N_t(f)$, is defined as $N_t(f) \equiv \left[\int_0^1 N_t(h,f)^{\frac{\zeta_{N,t}-1}{\zeta_{N,t}}} dh\right]^{\frac{\zeta_{N,t}}{\zeta_{N,t}-1}}$ where we suppress superscripts $L$ and $P$. Households are monopolistic suppliers of labor and set their wages flexibly by incorporating a markup over their utility cost of work. The wage index is derived by $W_t = \left[\int_0^1 W_t(h)^{1-\zeta_{N,t}} dh\right]^{\frac{1}{1-\zeta_{N,t}}}$. The elasticity of substitution between differentiated types of labor ($\zeta_{N,t}$) is assumed to be time-varying, and so is the wage markup, $\frac{1}{\zeta_{N,t}-1}$.[25] There are two sources of uncertainty from the supply side: a productivity shock $A_t$ and a labor-markup shock $\zeta_{N,t}$, which are common to all Home firms. Profits of LCP and PCP firms in Home are given by

$$\begin{aligned} \Pi_t^L(f) &= P_{Ht}^L(f)C_{Ht}^L(f) + \mathcal{E}_t P_{Ht}^{L*}(f)C_{Ht}^{L*}(f) - (1-\tau)W_t N_t^L(f), \\ \Pi_t^P(f) &= P_{Ht}^P(f)\left[C_{Ht}^P(f) + C_{Ht}^{P*}(f)\right] - (1-\tau)W_t N_t^P(f), \end{aligned} \tag{7}$$

where the law of one price holds for PCP products, $P_{Ht}^P(f) = \mathcal{E}_t P_{Ht}^{P*}(f)$. $\tau$ is a subsidy to Home firms which is imposed only for the efficiency of the steady state. Aggregate profits, $\Gamma_t \equiv \int_0^\chi \Pi_t^L(f)df + \int_\chi^1 \Pi_t^P(f)df$, are transferred to households who own the firms.

On a staggered basis as in Calvo (1983), PCP firms set a single price, $P_{Ht}^P(f)$, in their own currency, whereas LCP firms set nominal prices, $P_{Ht}^L(f)$ and $P_{Ht}^{L*}(f)$, in currency units that are local to where the good is sold. In each period $t$, a firm can reoptimize its nominal price with a constant probability $1-\theta$ and set a new price: $P_{Ht}^{P,o}(f)$ under PCP and $P_{Ht}^{L,o}(f)$ and $P_{Ht}^{L,o*}(f)$ under LCP. The opportunity for reoptimizing prices is granted independently across firms and time. By the law of large numbers, a fraction $\theta$ of firms maintains the same price from the previous period: $P_{H,t-1}^P(f)$, $P_{H,t-1}^L(f)$ and $P_{H,t-1}^{L*}(f)$. $C_{Ht}^L(f)$ and $C_{Ht}^{L*}(f)$ represent demands for products of a Home LCP firm $f$ from Home and Foreign markets, respectively. Therefore, total output of the LCP firm is given by $Y_t^L(f) \equiv C_{Ht}^L(f) + C_{Ht}^{L*}(f)$. Similarly, $Y_t^P(f) \equiv C_{Ht}^P(f) + C_{Ht}^{P*}(f)$ is total output of a

[25] To be clear, we assume exogenous variation in the wage markup only for the presence of a cost push shock in the Phillips curve.

Home PCP firm $f$.

LCP and PCP firms in Foreign price their products analogously, with profits given by $\Pi_t^{L*}(f^*)$ and $\Pi_t^{P*}(f^*)$, a shock to productivity $A_t^*$, a labor-markup shock $\zeta_{N,t}^*$ and the subsidy $\tau^*$. To ease discussion on the global loss function in the subsequent section 4.1, we display price indexes explicitly below. Price indexes for LCP and PCP products in Home and Foreign are defined as

$$
\begin{aligned}
&P_{Ht}^{L} \equiv \left[\tfrac{1}{\chi}\int_0^{\chi}\left(P_{Ht}^{L}(f)\right)^{1-\xi}df\right]^{\frac{1}{1-\xi}}, \qquad && P_{Ft}^{L*} \equiv \left[\tfrac{1}{\chi}\int_0^{\chi}\left(P_{Ft}^{L*}(f^*)\right)^{1-\xi}df^*\right]^{\frac{1}{1-\xi}}, \\
&P_{Ht}^{L*} \equiv \left[\tfrac{1}{\chi}\int_0^{\chi}\left(P_{Ht}^{L*}(f)\right)^{1-\xi}df\right]^{\frac{1}{1-\xi}}, && P_{Ft}^{L} \equiv \left[\tfrac{1}{\chi}\int_0^{\chi}\left(P_{Ft}^{L}(f^*)\right)^{1-\xi}df^*\right]^{\frac{1}{1-\xi}}, \\
&P_{Ht}^{P} \equiv \left[\tfrac{1}{1-\chi}\int_{\chi}^{1}\left(P_{Ht}^{P}(f)\right)^{1-\xi}df\right]^{\frac{1}{1-\xi}}, && P_{Ft}^{P*} \equiv \left[\tfrac{1}{1-\chi}\int_{\chi}^{1}\left(P_{Ft}^{P*}(f^*)\right)^{1-\xi}df^*\right]^{\frac{1}{1-\xi}}.
\end{aligned}
\tag{8}
$$

The notation $P$ with the subscript $H(F)$ indicates a nominal price of a product produced by a Home(Foreign) firm. $P_{Ht}^{L}(f)$ stands for the price of a Home LCP product $f$ sold in Home; $P_{Ht}^{L*}(f)$ denotes the price of a Home LCP product $f$ sold in Foreign; $P_{Ht}^{P}(f)$ is the price of a Home PCP product $f$ sold in Home and Foreign. $P_{Ft}^{L*}(f)$, $P_{Ft}^{L}(f)$ and $P_{Ft}^{P*}(f)$ are Foreign counterparts. Therefore, producer price indexes $(P_{Ht}, P_{Ft}, P_{Ft}^*, P_{Ht}^*)$ and consumer price indexes $(P_t, P_t^*)$ dual to (2) and (3) are derived by

$$
\begin{aligned}
P_{Ht} &= \left[\chi {P_{Ht}^{L}}^{1-\xi} + (1-\chi){P_{Ht}^{P}}^{1-\xi}\right]^{\frac{1}{1-\xi}}, & P_{Ft}^* &= \left[\chi {P_{Ft}^{L*}}^{1-\xi} + (1-\chi){P_{Ft}^{P*}}^{1-\xi}\right]^{\frac{1}{1-\xi}}, \\
P_{Ft} &= \left[\chi {P_{Ft}^{L}}^{1-\xi} + (1-\chi)\left(\mathcal{E}_t P_{Ft}^{P*}\right)^{1-\xi}\right]^{\frac{1}{1-\xi}}, & P_{Ht}^* &= \left[\chi {P_{Ht}^{L*}}^{1-\xi} + (1-\chi)\left(\frac{P_{Ht}^{P}}{\mathcal{E}_t}\right)^{1-\xi}\right]^{\frac{1}{1-\xi}}, \\
P_t &= \left[\tfrac{\nu}{2}{P_{Ht}}^{1-\epsilon} + \left(1-\tfrac{\nu}{2}\right){P_{Ft}}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, & P_t^* &= \left[\tfrac{\nu}{2}{P_{Ft}^*}^{1-\epsilon} + \left(1-\tfrac{\nu}{2}\right){P_{Ht}^*}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.
\end{aligned}
\tag{9}
$$

It is worth noting that these price indexes nest the two polar cases of LCP and PCP. Under full price rigidities, nominal exchange rate movements are completely mute in import prices under LCP ($\chi = 1$) while import price indexes move one-to-one with nominal exchange rate fluctuations under PCP ($\chi = 0$).

### 2.3 Equilibrium in Goods Markets

Using Hicksian demand functions for Home and Foreign goods from (2), we derive aggregate demand given by

$$
\begin{aligned}
Y_t \equiv C_{Ht} + C_{Ht}^* &= \tfrac{\nu}{2}\left(\tfrac{P_{Ht}}{P_t}\right)^{-\epsilon} C_t + \left(1-\tfrac{\nu}{2}\right)\left(\tfrac{P_{Ht}^*}{P_t^*}\right)^{-\epsilon} C_t^*, \\
Y_t^* \equiv C_{Ft}^* + C_{Ft} &= \tfrac{\nu}{2}\left(\tfrac{P_{Ft}^*}{P_t^*}\right)^{-\epsilon} C_t^* + \left(1-\tfrac{\nu}{2}\right)\left(\tfrac{P_{Ft}}{P_t}\right)^{-\epsilon} C_t.
\end{aligned}
\tag{10}
$$

Goods market clearing conditions relate total labor to aggregate demand and price dispersion terms:

$$
\begin{aligned}
A_t N_t &= \textstyle\int_0^\chi A_t N_t^L(f)df + \int_\chi^1 A_t N_t^P(f)df, \\
&= C_{Ht}\left(\chi V_{Ht}^L + (1-\chi)V_{Ht}^P\right) + C_{Ht}^*\left(\chi V_{Ht}^{L*} + (1-\chi)V_{Ht}^{P*}\right),
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
A_t^* N_t^* &= \textstyle\int_0^\chi A_t^* N_t^{L*}(f^*)df^* + \int_\chi^1 A_t^* N_t^{P*}(f^*)df^*, \\
&= C_{Ft}^*\left(\chi V_{Ft}^{L*} + (1-\chi)V_{Ft}^{P*}\right) + C_{Ft}\left(\chi V_{Ft}^L + (1-\chi)V_{Ft}^P\right),
\end{aligned}
\tag{12}
$$

where price dispersion terms for products of LCP and PCP firms in Home are defined as

$$
\begin{aligned}
V_{Ht}^L &\equiv \tfrac{1}{\chi}\textstyle\int_0^\chi \left(\tfrac{P_{Ht}^L(f)}{P_{Ht}}\right)^{-\xi} df, &\qquad V_{Ht}^{L*} &\equiv \tfrac{1}{\chi}\textstyle\int_0^\chi \left(\tfrac{P_{Ht}^{L*}(f)}{P_{Ht}^*}\right)^{-\xi} df, \\
V_{Ht}^P &\equiv \tfrac{1}{1-\chi}\textstyle\int_\chi^1 \left(\tfrac{P_{Ht}^P(f)}{P_{Ht}}\right)^{-\xi} df, &\qquad V_{Ht}^{P*} &\equiv \tfrac{1}{1-\chi}\textstyle\int_\chi^1 \left(\tfrac{\frac{1}{\mathcal{E}_t}P_{Ht}^P(f)}{P_{Ht}^*}\right)^{-\xi} df.
\end{aligned}
$$

Likewise, price dispersion terms in Foreign ($V_{Ft}^{L*}$, $V_{Ft}^L$, $V_{Ft}^{P*}$, $V_{Ft}^P$) are specified in the appendix A.

### 2.4 International Financial Markets

Due to capital market distortions, there is a wedge in the ex-post relation of intertemporal marginal rates of substitution between two countries. Optimal risk sharing under capital controls implies

$$
\left(\frac{C_t^*(h^*)}{C_t(h)}\right)^{-\sigma} = \left(\frac{\zeta_{C,t}}{\zeta_{C,t}^*}\right) Q_t\left(1+\varrho_t\right),
\tag{13}
$$

where $Q_t$ is the real exchange rate, defined as $Q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$, representing units of Home aggregate consumption per one unit of Foreign aggregate consumption. Following Corsetti et al. (2020), we define a measure of the degree of imperfect risk sharing as the cross-country utility value of Home aggregate consumption, $F_t$, given by

$$F_t \equiv \left( \frac{\zeta_{C,t}^* C_t^*(h^*)^{-\sigma}}{\zeta_{C,t} C_t(h)^{-\sigma}} \right) \left( \frac{1}{Q_t} \right) = 1 + \varrho_t. \tag{14}$$

Here the second equality holds from the relation (13). The equation (14) implies that $F_t$ represents the ratio of the Foreign household's subjective valuation on Home aggregate consumption to the Home household's. The Home household should consume $F_t$ units of Home aggregate consumption for him to be as well off as the Foreign household who receives one unit of Home aggregate consumption.[26] Without capital controls this ratio is unity ($F_t = 1$) and the international risk sharing is perfect. By contrast, non-unitary $F_t$ implies Home and Foreign households' valuations on the same composite consumption are imbalanced and the risk sharing is inefficient. For this reason, we label $F_t$ "demand imbalance"(Corsetti et al. (2020)). In equilibrium, demand imbalance equals a gross tax rate on net capital flow.

Finally, following Engel (2011), we define the Home(Foreign) relative price of imports $S_t(S_t^*)$, currency misalignment $M_t$ and export premium $Z_t$ as

$$\begin{aligned} &S_t \equiv \tfrac{P_{Ft}}{P_{Ht}}, && \text{and} \quad S_t^* \equiv \tfrac{P_{Ht}^*}{P_{Ft}^*}, \\ &M_t \equiv \left[ \left( \tfrac{\mathcal{E}_t P_{Ht}^*}{P_{Ht}} \right) \left( \tfrac{\mathcal{E}_t P_{Ft}^*}{P_{Ft}} \right) \right]^{\frac{1}{2}}, && \text{and} \quad Z_t \equiv \left[ \left( \tfrac{\mathcal{E}_t P_{Ht}^*}{P_{Ht}} \right) \left( \tfrac{\frac{1}{\mathcal{E}_t} P_{Ft}}{P_{Ft}^*} \right) \right]^{\frac{1}{2}} = [S_t S_t^*]^{\frac{1}{2}}. \end{aligned} \tag{15}$$

Here $S_t$ and $S_t^*$ represent the relative price of imported goods to domestically produced goods.[27] Currency misalignment is a measure of the average discrepancy in prices of identical goods between two countries, that is, the degree of deviation from the law of one price. Export premium measures the average difference between exporting and local prices of identical goods in two countries.[28]

[26] That is, $F_t \zeta_{C,t} C_t(h)^{-\sigma} = \frac{\zeta_{C,t}^* C_t^*(h^*)^{-\sigma}}{Q_t}$ holds. Consuming one unit of Home aggregate consumption, the Home household derives $\zeta_{C,t} C_t(h)^{-\sigma}$ utils, whereas the Foreign household obtains $\frac{\zeta_{C,t}^* C_t^*(h^*)^{-\sigma}}{Q_t}$ utils by exchanging it for $\frac{1}{Q_t}$ units of Foreign aggregate consumption.

[27] Note that these are different from the terms of trade which represent the price of imported goods relative to exported goods sold in export markets. The terms of trade are defined as $\mathcal{T}_t = \frac{P_{Ft}}{\mathcal{E}_t P_{Ht}^*}$ for Home and $\mathcal{T}_t^* = \frac{\mathcal{E}_t P_{Ht}^*}{P_{Ft}}$ for Foreign. Instead of using the terms of trade, we proceed with the price of imported goods relative to domestically-produced goods sold in domestic markets. Using $S_t$ and $S_t^*$ facilitates the algebraic analysis.

[28] Note that $\mathcal{E}_t P_{Ht}^*$ denotes the Home-currency price of Home-produced goods sold in Foreign markets; $\mathcal{E}_t P_{Ft}^*$ denotes the Home-currency price of Foreign-produced goods sold in Foreign markets; and $\frac{1}{\mathcal{E}_t} P_{Ft}$ represents the Foreign-currency price of Foreign-produced goods sold in Home markets. In the log-linearized model, we define "relative" and "world" values for any variables $x_t$ and $x_t^*$ as $x_t^R \equiv \frac{x_t - x_t^*}{2}$ and $x_t^W \equiv \frac{x_t + x_t^*}{2}$. Observe that currency misalignment is the world value of price misalignments between the two countries and export premium is the relative value of price misalignments. By price misalignment, we mean $\frac{\mathcal{E}_t P_{Ht}^*}{P_{Ht}}$ for Home goods and $\frac{\mathcal{E}_t P_{Ft}^*}{P_{Ft}}$ for Foreign goods. These are not unity if the law of one price does not hold.

Every detail of total equilibrium conditions is relegated to the online appendix A.

## 3 Log-Linearized Model

Throughout the paper, equilibrium conditions will be presented in the log-linear form approximated around the efficient nonstochastic steady state. As noted in section 2.2, we assume that the fiscal authority provides subsidies($\tau, \tau^*$) for firms, which correct steady-state markup distortions. The online appendix F contains the derivation of the log-linearized model. In what follows, a lowercase variable refers to the *log* deviation of the corresponding uppercase variable around its efficient steady state value.

As is standard in the Open-Economy New Keynesian framework, the system of equilibrium conditions consists of four components: aggregate demand, conditions implied by the structure of financial markets, aggregate supply represented by inflation adjustments and monetary policy. Since we focus on targeting rules, not instrument rules, conditions for optimal monetary policy will be derived in the subsequent section 4. Before proceeding, we first specify the relative price of imported goods ($s_t, s_t^*$) and currency misalignment ($m_t$) in a log-linear form. The first-order log-linearized export premium ($z_t$) turns out to be zero under our environment which features both financial market distortions and generic degree of exchange rate pass-through.[29] Making use of $z_t = 0$, we can simplify currency misalignment and relative prices to

$$m_t = e_t + p_{Ht}^* - p_{Ht} = e_t + p_{Ft}^* - p_{Ft} \quad \text{and} \quad s_t = p_{Ft} - p_{Ht} = p_{Ft}^* - p_{Ht}^* = -s_t^*, \tag{16}$$

where $e_t$ denotes the nominal exchange rate in log. Equations for aggregate demand involve the representative household's euler equation and market demand on Home and Foreign goods, given

[29]This is not surprising since Engel (2011) shows export premium is zero under full LCP. Since there is no cross-country price misalignment under full PCP, it is obvious that the export premium is also zero in the economy with a mixture of PCP and LCP. See the online appendix F.2: Proof of Zero Export Premium ($z_t = 0\ \forall t$) in Equilibrium.

by

$$
\begin{aligned}
&(Home) \quad \begin{aligned} i_t &= \sigma\left(\mathbb{E}_t c_{t+1} - c_t\right) + \mathbb{E}_t \pi_{t+1} - \mathbb{E}_t \zeta_{c,t+1} + \zeta_{c,t}, \\ y_t &= \left(\tfrac{\nu}{2}\right) c_t + \left(1 - \tfrac{\nu}{2}\right) c_t^* + \epsilon\left(\tfrac{\nu}{2}\right)\left(1 - \tfrac{\nu}{2}\right)\left(s_t - s_t^*\right), \end{aligned} \\
&(Foreign) \quad \begin{aligned} i_t^* &= \sigma\left(\mathbb{E}_t c_{t+1}^* - c_t^*\right) + \mathbb{E}_t \pi_{t+1}^* - \mathbb{E}_t \zeta_{c,t+1}^* + \zeta_{c,t}^*, \\ y_t^* &= \left(\tfrac{\nu}{2}\right) c_t^* + \left(1 - \tfrac{\nu}{2}\right) c_t - \epsilon\left(\tfrac{\nu}{2}\right)\left(1 - \tfrac{\nu}{2}\right)\left(s_t - s_t^*\right), \end{aligned}
\end{aligned} \tag{17}
$$

where preference shocks are redefined as $\zeta_{c,t} \equiv \log\left(\zeta_{C,t}\right)$ and $\zeta_{c,t}^* \equiv \log\left(\zeta_{C,t}^*\right)$ in log. Aggregate demand equates to total output through goods market clearing conditions: $y_t = a_t + n_t$ and $y_t^* = a_t^* + n_t^*$. These equations imply that a short-term nominal interest rate $i_t(i_t^*)$ is inversely associated with aggregate output $y_t(y_t^*)$, which represents the "intertemporal IS relation." $\pi_t(\pi_t^*)$ denotes CPI inflation in Home(Foreign) defined as

$$
\pi_t \equiv \log\left(\frac{P_t}{P_{t-1}}\right) = \left(\tfrac{\nu}{2}\right)\pi_{Ht} + \left(1 - \tfrac{\nu}{2}\right)\pi_{Ft} \quad \text{and} \quad \pi_t^* \equiv \log\left(\frac{P_t^*}{P_{t-1}^*}\right) = \left(\tfrac{\nu}{2}\right)\pi_{Ft}^* + \left(1 - \tfrac{\nu}{2}\right)\pi_{Ht}^*,
$$

where PPI inflation rates are

$$
\pi_{Ht} \equiv \log\left(\frac{P_{Ht}}{P_{H,t-1}}\right), \quad \pi_{Ft} \equiv \log\left(\frac{P_{Ft}}{P_{F,t-1}}\right), \quad \pi_{Ft}^* \equiv \log\left(\frac{P_{Ft}^*}{P_{F,t-1}^*}\right), \quad \text{and} \quad \pi_{Ht}^* \equiv \log\left(\frac{P_{Ht}^*}{P_{H,t-1}^*}\right).
$$

Since it is convenient to use "relative" and "world" values in characterizing optimal cooperative monetary policy, we define $x_t^R \equiv \frac{x_t - x_t^*}{2}$ and $x_t^W \equiv \frac{x_t + x_t^*}{2}$ for any variables $x_t$ and $x_t^*$. Equilibrium conditions arising from imperfect international capital markets are represented by

$$
\sigma(c_t - c_t^*) = (\nu - 1)s_t + m_t + f_t + \zeta_{c,t} - \zeta_{c,t}^*, \tag{18}
$$

$$
\begin{aligned}
f_t &= \left(\frac{\lambda}{1-\lambda}\right)\left(1 - \frac{\nu}{2}\right)\left[q_t - (c_t - c_t^*) + \nu(\epsilon - 1)\left(\frac{s_t - s_t^*}{2}\right)\right] \\
&= \frac{2\lambda(2-\nu)[\sigma(\epsilon\nu - 1) + 1 - \nu]}{\lambda(2-\nu)(\epsilon\nu - \nu + 1) + 2(1-\lambda)D} y_t^R - \frac{\lambda(2-\nu)(\epsilon\nu - \nu + 1 - D)}{\lambda(2-\nu)(\epsilon\nu - \nu + 1) + 2(1-\lambda)D} m_t - \frac{\lambda(2-\nu)(\epsilon\nu - \nu + 1)}{\lambda(2-\nu)(\epsilon\nu - \nu + 1) + 2(1-\lambda)D}\left(\zeta_{c,t} - \zeta_{c,t}^*\right),
\end{aligned} \tag{19}
$$

where the real exchange rate is given by $q_t \equiv e_t + p_t^* - p_t = (\nu - 1)s_t + m_t$ and a constant $D$ is defined as $D \equiv (\nu - 1)^2 + \sigma\epsilon\nu(2 - \nu)$. Combined with the IS relation (17), the risk sharing condition

(18) can be rewritten out in terms of cross-country output difference as

$$s_t = \frac{2\sigma}{D} y_t^R - \frac{\nu - 1}{D}(m_t + f_t + \zeta_{c,t} - \zeta_{c,t}^*). \tag{20}$$

It is important to observe that if we set $\sigma = \epsilon = 1$, relative output($y_t^R$) and currency misalignment($m_t$) in (19) vanish and the financial market condition (19) reduces to

$$f_t = \left(\tfrac{-\lambda(2-\nu)}{2-\lambda\nu}\right)\left(\zeta_{c,t} - \zeta_{c,t}^*\right).$$

This equation replicates the well-known Cole and Obstfeld (1991) result when the economy is disturbed by supply shocks. That is, the degree of imperfect financial integration($\lambda$) does not affect the degree of cross-country risk sharing in response to a productivity shock($a_t$) or a labor-markup shock($\zeta_{N,t}$). However, if the world economy is shocked asymmetrically from the demand side($\zeta_{c,t} - \zeta_{c,t}^* \neq 0$), then cross-county risk sharing is imperfect($f_t \neq 0$) and financial market frictions impinge on equilibrium allocations even with $\sigma = \epsilon = 1$. Formally, we provide two propositions in order to establish the irrelevance result of financial market structure for international risk sharing in the context of our model for optimal monetary policy.[30]

**Proposition 1.** *In response to asymmetric preference shocks* $\left(\zeta_{c,t} \neq \zeta_{c,t}^*\right)$*, the risk sharing is always imperfect if international financial markets are frictional* ($\lambda > 0$) *and goods markets in Home and Foreign are not completely separated* ($\nu \neq 2$)*.*

**Proposition 2.** *Suppose the cross-country trade elasticity is unity* ($\epsilon = 1$) *and consumption utility is in log* ($\sigma = 1$)*. Then the degree of cross-country risk sharing is determined solely by the cross-country difference of preference shocks through* $f_t = -\left(\zeta_{c,t} - \zeta_{c,t}^*\right)\left(\tfrac{\lambda(2-\nu)}{2-\nu\lambda}\right)$*. If shocks to preference are symmetric between Home and Foreign* $\left(\zeta_{c,t} = \zeta_{c,t}^*\right)$*, the cross-country risk sharing is perfect.*

The study of optimal cooperative monetary policy under generic degrees of exchange rate pass-through($\chi$) and cross-country financial integration($\lambda$) is the key element in our analysis which departs from the previous studies. Note that perfect financial integration combined with PCP($\lambda = \chi = 0$) yields simpler expression for (18): $\sigma(c_t - c_t^*) = (\nu - 1)s_t + \zeta_{c,t} - \zeta_{c,t}^*$. Currency misalignment($m_t$) is added to this risk-sharing condition (18) since identical goods might be sold at different prices across countries as in Engel (2011). Demand imbalance($f_t$) also creates an additional wedge in

[30]Formal proofs are available in the online appendix F.5.

the equation (18) because capital controls distort Home and Foreign households' valuations on aggregate consumption as in Corsetti et al. (2020).

Home and Foreign households' wage setting conditions can be approximated as

$$
\begin{aligned}
(Home) \quad & w_t - p_{Ht} = \sigma c_t + \phi n_t + \left(1 - \tfrac{\nu}{2}\right) s_t + \tfrac{1}{\delta} u_t - \zeta_{c,t}, \\
(Foreign) \quad & w_t^* - p_{Ft}^* = \sigma c_t^* + \phi n_t^* + \left(1 - \tfrac{\nu}{2}\right) s_t^* + \tfrac{1}{\delta} u_t^* - \zeta_{c,t}^*,
\end{aligned} \tag{21}
$$

where $\delta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Here we redefine shocks to the labor markup as

$$
u_t \equiv \delta \left[\log\left(\frac{\zeta_{N,t}}{\zeta_{N,t}-1}\right) - \log\left(\frac{\zeta_N}{\zeta_N - 1}\right)\right] \quad \text{and} \quad u_t^* \equiv \delta \left[\log\left(\frac{\zeta_{N,t}^*}{\zeta_{N,t}^*-1}\right) - \log\left(\frac{\zeta_N^*}{\zeta_N^* - 1}\right)\right],
$$

which are usually interpreted as cost-push shocks in the literature. In the log-linear form of wage setting conditions, government subsidies offset steady-state distortions stemming from monopolies in labor and goods markets by $1 = (1-\tau)\left(\frac{\zeta_N}{\zeta_N-1}\right)\left(\frac{\xi}{\xi-1}\right) = (1-\tau^*)\left(\frac{\zeta_N^*}{\zeta_N^*-1}\right)\left(\frac{\xi}{\xi-1}\right)$.

In turn, we can derive log-linearized New Keynesian Phillips curves for open economies:

$$
\begin{aligned}
\pi_{Ht} \quad &= \quad \delta\left(w_t - p_{Ht} - a_t\right) + \beta \mathbb{E}_t\left[\pi_{H,t+1}\right], \\
\pi_{Ht}^* + (1-\chi)(e_t - e_{t-1}) \quad &= \quad \delta\left(w_t - p_{Ht} - a_t - m_t\right) + \beta \mathbb{E}_t\left[\pi_{H,t+1}^* + (1-\chi)(e_{t+1} - e_t)\right], \\
\pi_{Ft}^* \quad &= \quad \delta\left(w_t^* - p_{Ft}^* - a_t^*\right) + \beta \mathbb{E}_t\left[\pi_{F,t+1}^*\right], \\
\pi_{Ft} - (1-\chi)(e_t - e_{t-1}) \quad &= \quad \delta\left(w_t^* - p_{Ft}^* - a_t^* + m_t\right) + \beta \mathbb{E}_t\left[\pi_{F,t+1} - (1-\chi)(e_{t+1} - e_t)\right].
\end{aligned} \tag{22}
$$

It is worth noting that the expression (22) encompasses the polar cases of LCP and PCP. In LCP with $\chi = 1$, local price inflation of imported goods($\pi_{Ft}, \pi_{Ht}^*$) does not respond one-for-one to the nominal exchange rate and exporters bear the exchange rate risk.[31] By contrast, in PCP with $\chi = 0$, local price inflation of imported goods($\pi_{Ft}, \pi_{Ht}^*$) moves one-to-one with the nominal exchange rate growth and consumers take the exchange rate risk. Since there is no currency misalignment under PCP ($\chi = 0$), aggregate supply relations (22) reduce to two equations for Home and Foreign local price inflation rates, $\pi_{Ht}$ and $\pi_{Ft}^*$. By adjusting the parameter $\chi$ from zero to one, the model

[31] Under the Calvo-pricing friction, short-run exchange rate pass-through is not 0% in full LCP with $\chi = 1$. There are a portion of LCP firms who are allowed to reset their prices and nominal exchange rate movements affect import price inflation through currency misalignment ($m_t = e_t + p_{Ht}^* - p_{Ht} = e_t + p_{Ft}^* - p_{Ft}$) with the weight $\delta$. $\delta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$ is set to 0.0858 at the quarterly frequency as in the standard literature.

captures varying degrees of exchange rate pass-through from full LCP to full PCP.

Substituting out for real wages and productivity shocks, we can rewrite the Phillips curves in terms of a measure of the gap between actual output and efficient output, currency misalignment, demand imbalance and cost-push shocks, given by

$$\begin{aligned}
\pi_{Ht} &= \delta\left[\left(\tfrac{\sigma}{D}+\phi\right)\widetilde{y}_t^R + (\sigma+\phi)\widetilde{y}_t^W + \tfrac{D-\nu+1}{2D}(m_t+f_t)\right] + \beta\mathbb{E}_t\left[\pi_{H,t+1}\right] + u_t, \\
\pi_{Ft} - (1-\chi)\Delta e_t &= \begin{pmatrix} & \delta\left[\left(-\tfrac{\sigma}{D}-\phi\right)\widetilde{y}_t^R + (\sigma+\phi)\widetilde{y}_t^W + \tfrac{D+\nu-1}{2D}m_t + \tfrac{-D+\nu-1}{2D}f_t\right] \\ + & \beta\mathbb{E}_t\left[\pi_{F,t+1} - (1-\chi)\left(e_{t+1}-e_t\right)\right] + u_t^* \end{pmatrix}, \\
\pi_{Ft}^* &= \delta\left[\left(-\tfrac{\sigma}{D}-\phi\right)\widetilde{y}_t^R + (\sigma+\phi)\widetilde{y}_t^W + \tfrac{-D+\nu-1}{2D}(m_t+f_t)\right] + \beta\mathbb{E}_t\left[\pi_{F,t+1}^*\right] + u_t^*, \\
\pi_{Ht}^* + (1-\chi)\Delta e_t &= \begin{pmatrix} & \delta\left[\left(\tfrac{\sigma}{D}+\phi\right)\widetilde{y}_t^R + (\sigma+\phi)\widetilde{y}_t^W + \tfrac{-D-\nu+1}{2D}m_t + \tfrac{D-\nu+1}{2D}f_t\right] \\ + & \beta\mathbb{E}_t\left[\pi_{H,t+1}^* + (1-\chi)\left(e_{t+1}-e_t\right)\right] + u_t \end{pmatrix},
\end{aligned} \tag{23}$$

where $\Delta$ denotes the first difference operator. Here we define the gap of a variable $x_t$ from its efficient counterpart $\overline{x}_t$ as $\widetilde{x}_t \equiv x_t - \overline{x}_t$.[32]

Finally, following the tradition of the open-macro literature, it is natural to express the Phillips curves in cross-country sum and difference. If we use the relative and world CPI inflation rates and the relative price of imported goods, the four aggregate-supply relations (23) translate into three equations for Phillips curves: one for the cross-country difference between Home and Foreign CPI inflation rates given by

$$\begin{pmatrix} & \pi_t^R\left[1-(2-\nu)(1-\chi)\right] \\ - & \Delta m_t\left(1-\tfrac{\nu}{2}\right)(1-\chi) \\ - & \Delta s_t(\nu-1)\left(1-\tfrac{\nu}{2}\right)(1-\chi) \end{pmatrix} = \begin{pmatrix} & \delta\left[\left(\tfrac{\sigma}{D}+\phi\right)(\nu-1)\widetilde{y}_t^R + \left(\tfrac{D-\nu+1}{2D}\right)(\nu-1)f_t + \tfrac{D-(\nu-1)^2}{2D}m_t\right] \\ + & \beta\mathbb{E}_t\begin{bmatrix} & \pi_{t+1}^R\left[1-(2-\nu)(1-\chi)\right] \\ - & \Delta m_{t+1}\left(1-\tfrac{\nu}{2}\right)(1-\chi) \\ - & \Delta s_{t+1}(\nu-1)\left(1-\tfrac{\nu}{2}\right)(1-\chi) \end{bmatrix} + (\nu-1)\tfrac{u_t-u_t^*}{2} \end{pmatrix}, \tag{24}$$

another for the cross-country sum of Home and Foreign CPI inflation rates given by

$$\pi_t^W = \delta(\sigma+\phi)\widetilde{y}_t^W + \beta\mathbb{E}_t\left[\pi_{t+1}^W\right] + \tfrac{u_t+u_t^*}{2}, \tag{25}$$

[32] The details of log-linearized efficient allocations are presented in the online appendix F.6: Log-Linearized Globally Efficient Allocations.

and the third equation for the relative price of imported goods in Home given by

$$\begin{pmatrix} & \Delta s_t \left[1-(1-\chi)(\nu-1)\right] \\ - & (1-\chi)\left(\Delta m_t + 2\pi_t^R\right) \end{pmatrix} = \begin{pmatrix} -\delta\left[s_t - \overline{s}_t + 2\phi\widetilde{y}_t^R + f_t\right] \\ +\beta\mathbb{E}_t \begin{bmatrix} & \Delta s_{t+1}\left(1-(1-\chi)(\nu-1)\right) \\ - & (1-\chi)\left(\Delta m_{t+1} + 2\pi_{t+1}^R\right) \end{bmatrix} - (u_t - u_t^*) \end{pmatrix}, \tag{26}$$

where $\pi_t^R \equiv \frac{\pi_t - \pi_t^*}{2}$ and $\pi_t^W \equiv \frac{\pi_t + \pi_t^*}{2}$.

Together pricing-to-market, capital market imperfection and cost-push shocks give rise to a tension among the goals of zero inflation, output gap stabilization, zero demand imbalance and zero price dispersion among goods traded across countries. The policymaker seeks to find the optimal state-contingent evolution of macroeconomic variables to balance these goals, neither of which can be given absolute priority. In the next section, we discuss the policymaker's loss function under these frictions and characterize optimal monetary policy.

# 4 Optimal Monetary Policy

## 4.1 The Loss Function of Central Banks under Cooperation

Our aim is to characterize optimal cooperative monetary policy under commitment from a "timeless perspective." Benevolent central banks in two countries cooperate to maximize global welfare defined as an equally weighted average of the utilities of Home and Foreign households. We solve a linear-quadratic(LQ) approximate problem which consists of a quadratic objective and linear constraints and derive optimal targeting rules.[33] For analytical simplicity, monopolistic distortions are neutralized by appropriately chosen subsidies.

A second-order Taylor expansion of the periodic world welfare delivers the following loss function under financial market imperfection, $\lambda \in [0, 1]$ and generic degree of exchange rate pass-through,

[33] Linear targeting rules that close the system of equilibrium conditions are local linear approximations to the actual nonlinear optimal policy. As is well noted in Woodford (2003), this approach is valid as long as (i) structural shocks are small enough; (ii) in the absence of disturbances, approximated policies produce values that are close enough to the allocation around which Taylor-series expansions are taken; (iii) distortions are small enough. For the requirements of (ii) and (iii), a long-run average inflation rate is zero in our model and the market power of monopolists in the steady state is eliminated through the imposition of government subsidies. For a LQ problem in which conditions (ii) and (iii) are relaxed, see Benigno and Woodford (2005) and Benigno and Woodford (2012) for theoretical foundation and see Benigno and Benigno (2006) and De Paoli (2009) for the application to open economies.

$\chi \in [0,1]$[34]:

$$
L_t = \begin{pmatrix} + & \left(\frac{\sigma}{D}+\phi\right)\left(\widetilde{y}_t^R\right)^2 + (\sigma+\phi)\left(\widetilde{y}_t^W\right)^2 + \frac{\epsilon\nu(2-\nu)}{4D}\left(m_t+f_t\right)^2 \\ + & \left(\frac{\xi}{2}\right)\left[\left(\frac{\nu}{2}\right)\sigma_{P_H,t}{}^2 + \left(1-\frac{\nu}{2}\right)\sigma_{P_H^*,t}{}^2 + \left(\frac{\nu}{2}\right)\sigma_{P_F^*,t}{}^2 + \left(1-\frac{\nu}{2}\right)\sigma_{P_F,t}{}^2\right] \end{pmatrix}
$$

$$
= \begin{pmatrix} +\left(\frac{\sigma}{D}+\phi\right)\left(\widetilde{y}_t^R\right)^2 + (\sigma+\phi)\left(\widetilde{y}_t^W\right)^2 + \frac{\epsilon\nu(2-\nu)}{4D}\left(m_t+f_t\right)^2 \\ +\left(\frac{\xi}{2\delta}\right)\left[\ \left(\frac{\nu}{2}\right)(\pi_{H,t})^2 + \left(1-\frac{\nu}{2}\right)(\pi_{H,t}^*)^2 + \left(\frac{\nu}{2}\right)(\pi_{F,t}^*)^2 + \left(1-\frac{\nu}{2}\right)(\pi_{F,t})^2\ \right] \\ +\left(\frac{\xi}{2\delta}\right)\left[\begin{array}{ll} +\left(\frac{\nu}{2}\right) & \left(\frac{\chi}{\theta}\left\{\pi_{H,t}^L-\theta\pi_{H,t-1}^L\right\}^2 + \frac{1-\chi}{\theta}\left\{\pi_{H,t}^P-\theta\pi_{H,t-1}^P\right\}^2\right) \\ +\left(1-\frac{\nu}{2}\right) & \left(\frac{\chi}{\theta}\left\{\pi_{H,t}^{L*}-\theta\pi_{H,t-1}^{L*}\right\}^2 + \frac{1-\chi}{\theta}\left\{\pi_{H,t}^P-m_t-\theta(\pi_{H,t-1}^P-m_{t-1})\right\}^2\right) \\ +\left(\frac{\nu}{2}\right) & \left(\frac{\chi}{\theta}\left\{\pi_{F,t}^{L*}-\theta\pi_{F,t-1}^{L*}\right\}^2 + \frac{1-\chi}{\theta}\left\{\pi_{F,t}^{P*}-\theta\pi_{F,t-1}^{P*}\right\}^2\right) \\ +\left(1-\frac{\nu}{2}\right) & \left(\frac{\chi}{\theta}\left\{\pi_{F,t}^L-\theta\pi_{F,t-1}^L\right\}^2 + \frac{1-\chi}{\theta}\left\{\pi_{F,t}^{P*}+m_t-\theta(\pi_{F,t-1}^{P*}+m_{t-1})\right\}^2\right) \\ +\left(1-\frac{\nu}{2}\right) & (1-\chi)\begin{pmatrix} +(\Delta e_t)^2 \\ +\begin{pmatrix} & \pi_{H,t}^P-\pi_{H,t-1}^P-\pi_{F,t}^{P*}+\pi_{F,t-1}^{P*} \\ + & \pi_{H,t}^*-\pi_{F,t}-2m_t+2m_{t-1}+\Delta e_t \end{pmatrix}^2 \\ -\begin{pmatrix} & \pi_{H,t}^P-\pi_{H,t-1}^P-\pi_{F,t}^{P*}+\pi_{F,t-1}^{P*} \\ + & \pi_{H,t}^*-\pi_{F,t}-2m_t+2m_{t-1} \end{pmatrix}^2 \end{pmatrix} \end{array}\right] \end{pmatrix}, \tag{27}
$$

where $\sigma^2_{P_H,t}$, $\sigma^2_{P_H^*,t}$, $\sigma^2_{P_F^*,t}$ and $\sigma^2_{P_F,t}$ are measures of price dispersion across firms. The nominal exchange rate growth and the cross-country difference of PPI inflation of imported goods can be substituted out by $\Delta e_t = \Delta m_t + (\nu-1)\Delta s_t + 2\pi_t^R$ and $\Delta e_t + \pi_{Ht}^* - \pi_{Ft} = \Delta m_t - \Delta s_t$.[35] The weighted sum of PPI inflation rates can be expressed in terms of cross-country CPI inflation and the relative price of imported goods by

$$
\left(\frac{\nu}{2}\right)(\pi_{H,t})^2 + \left(1-\frac{\nu}{2}\right)(\pi_{H,t}^*)^2 + \left(\frac{\nu}{2}\right)(\pi_{F,t}^*)^2 + \left(1-\frac{\nu}{2}\right)(\pi_{F,t})^2 = 2\left(\pi_t^R\right)^2 + 2\left(\pi_t^W\right)^2 + \frac{\nu(2-\nu)}{2}\left(\Delta s_t\right)^2.
$$

In the global loss function (27), we deflate all price indexes for LCP and PCP products in (8) by relevant producer price indexes:

$$
\begin{aligned}
\pi_{Ht}^L &\equiv \log\left(\frac{P_{Ht}^L}{P_{Ht}}\right), \quad \pi_{Ht}^{L*} \equiv \log\left(\frac{P_{Ht}^{L*}}{P_{Ht}^*}\right), \quad \pi_{Ht}^P \equiv \log\left(\frac{P_{Ht}^P}{P_{Ht}}\right), \\
\pi_{Ft}^{L*} &\equiv \log\left(\frac{P_{Ft}^{L*}}{P_{Ft}^*}\right), \quad \pi_{Ft}^L \equiv \log\left(\frac{P_{Ft}^L}{P_{Ft}}\right), \quad \pi_{Ft}^{P*} \equiv \log\left(\frac{P_{Ft}^{P*}}{P_{Ft}^*}\right).
\end{aligned}
$$

Note that after linearization, equations for producer price indexes in (9) can be rewritten in terms

[34] The derivation of (27) is displayed in the online appendix G.

[35] For the derivation, we refer to the online appendix F.4: The Relationship between CPI and PPI Inflation Rates.

of deflated LCP and PCP price indexes as

$$
\begin{aligned}
0 &= \chi\pi_{Ht}^{L} + (1-\chi)\pi_{Ht}^{P}, \qquad 0 = \chi\pi_{Ht}^{L*} + (1-\chi)\left(\pi_{Ht}^{P} - m_t\right), \\
0 &= \chi\pi_{Ft}^{L*} + (1-\chi)\pi_{Ft}^{P*}, \qquad 0 = \chi\pi_{Ft}^{L} + (1-\chi)\left(\pi_{Ft}^{P*} + m_t\right).
\end{aligned} \tag{28}
$$

The global loss (27) generalizes loss functions in Engel (2011) and Corsetti et al. (2020) to varying degrees of exchange rate pass-through, $\chi \in [0,1]$. In LCP ($\chi = 1$), the relation (28) implies deflated LCP price indexes are fixed at zero and the global loss translates into

$$
L_t^{LCP} = \begin{pmatrix} +\left(\frac{\sigma}{D}+\phi\right)\left(\widetilde{y}_t^R\right)^2 + (\sigma+\phi)\left(\widetilde{y}_t^W\right)^2 + \frac{\epsilon\nu(2-\nu)}{4D}\left(m_t+f_t\right)^2 \\ +\left(\frac{\xi}{2\delta}\right)\left[2\left(\pi_t^R\right)^2 + 2\left(\pi_t^W\right)^2 + \frac{\nu(2-\nu)}{2}\left(\Delta s_t\right)^2\right] \end{pmatrix}. \tag{29}
$$

On the other hand, in PCP ($\chi = 0$), deflated PCP price indexes and currency misalignment are all zero and the global loss reduces to

$$
\begin{aligned}
L_t^{PCP} &= \begin{pmatrix} +\left(\frac{\sigma}{D}+\phi\right)\left(\widetilde{y}_t^R\right)^2 + (\sigma+\phi)\left(\widetilde{y}_t^W\right)^2 + \frac{\epsilon\nu(2-\nu)}{4D}\left(m_t+f_t\right)^2 \\ +\left(\frac{\xi}{2\delta}\right)\left[\ \left(\frac{\nu}{2}\right)(\pi_{H,t})^2 + \left(1-\frac{\nu}{2}\right)(\pi_{H,t}^* + \Delta e_t)^2 + \left(\frac{\nu}{2}\right)(\pi_{F,t}^*)^2 + \left(1-\frac{\nu}{2}\right)(\pi_{F,t} - \Delta e_t)^2\ \right] \end{pmatrix} \\
&= \left(\frac{\sigma}{D}+\phi\right)\left(\widetilde{y}_t^R\right)^2 + (\sigma+\phi)\left(\widetilde{y}_t^W\right)^2 + \frac{\epsilon\nu(2-\nu)}{4D}\left(f_t\right)^2 + \left(\frac{\xi}{2\delta}\right)\left[(\pi_{H,t})^2 + (\pi_{F,t}^*)^2\right],
\end{aligned} \tag{30}
$$

where the last equality follows from the law of one price: $m_t = 0$, $\pi_{Ht}^* + \Delta e_t = \pi_{Ht}$ and $\pi_{Ft}^* = \pi_{Ft} - \Delta e_t$.[36] The loss functions (29) and (30) exactly replicate those derived in Engel (2011) and Corsetti et al. (2020).

It is apparent from (27) that the coexistence of LCP and PCP products captured by $\chi \in [0,1]$ adds more dispersion terms to the cross-section variances of consumer prices, $\sigma^2_{P_H,t}$, $\sigma^2_{P_H^*,t}$, $\sigma^2_{P_F^*,t}$ and $\sigma^2_{P_F,t}$. To gain intuition, it is useful to rewrite the weighted sum of cross-section variances of

[36] It is worth noting that $L_t^{LCP}$ in (29) does not subsume $L_t^{PCP}$ in (30). To see this, note that $\pi_{Ht} = \pi_t^W + \pi_t^R - \left(1-\frac{\nu}{2}\right)\Delta s_t$ and $\pi_{Ft}^* = \pi_t^W - \pi_t^R + \left(1-\frac{\nu}{2}\right)\Delta s_t$ hold (see the online appendix F.4). Therefore, we obtain $(\pi_{H,t})^2 + (\pi_{F,t}^*)^2 = 2\left(\pi_t^R - \left(1-\frac{\nu}{2}\right)\Delta s_t\right)^2 + 2\left(\pi_t^W\right)^2$, which implies $L_t^{LCP}$ does not translate into $L_t^{PCP}$ even with $m_t = 0$ imposed.

consumer prices as[37]

$$
\begin{aligned}
&\left(\tfrac{\nu}{2}\right)\sigma_{P_H,t}{}^2+\left(1-\tfrac{\nu}{2}\right)\sigma_{P_H^*,t}{}^2+\left(\tfrac{\nu}{2}\right)\sigma_{P_F^*,t}{}^2+\left(1-\tfrac{\nu}{2}\right)\sigma_{P_F,t}{}^2\\
=\;&\left(\tfrac{1}{\delta}\right)\left[\begin{array}{ll}
 & +\left(\tfrac{\nu}{2}\right)(\pi_{H,t})^2+\left(1-\tfrac{\nu}{2}\right)(\pi^*_{H,t})^2+\left(\tfrac{\nu}{2}\right)(\pi^*_{F,t})^2+\left(1-\tfrac{\nu}{2}\right)(\pi_{F,t})^2\\
+\left(\tfrac{\nu}{2}\right) & \tfrac{\chi(1-\chi)(1-\theta)^2}{\theta}\left[\pi^{L,o}_{Ht}-\pi^{P,o}_{Ht}\right]^2+\left(\tfrac{\nu}{2}\right)\tfrac{\chi(1-\chi)(1-\theta)^2}{\theta}\left[\pi^{L,o*}_{Ft}-\pi^{P,o*}_{Ft}\right]^2\\
+\left(1-\tfrac{\nu}{2}\right) & \tfrac{\chi(1-\chi)(1-\theta)^2}{\theta}\left((1-\chi)\left[\pi^{L,o*}_{Ht}-\pi^{P,o}_{Ht}+m_t\right]^2+\chi\left[\pi^{L,o}_{Ht}-\pi^{P,o}_{Ht}\right]^2\right)\\
+\left(1-\tfrac{\nu}{2}\right) & \tfrac{\chi(1-\chi)(1-\theta)^2}{\theta}\left((1-\chi)\left[\pi^{L,o}_{Ft}-\pi^{P,o*}_{Ft}-m_t\right]^2+\chi\left[\pi^{L,o*}_{Ft}-\pi^{P,o*}_{Ft}\right]^2\right)\\
+\left(1-\tfrac{\nu}{2}\right) & \left(\begin{array}{l}
+(1-\chi)\left[1+2\theta\chi(1-\chi)\right](e_t-e_{t-1})^2+\tfrac{(1-\chi)}{\theta}(m_t-\theta m_{t-1})^2\\
+(1-\chi)\left(\begin{array}{l}
\chi(1-\chi)(1-\theta)\left(\pi^{L,o*}_{Ht}-\pi^{P,o}_{Ht}-\pi^{L,o}_{Ft}+\pi^{P,o*}_{Ft}+2m_t\right)\\
+\pi^P_{H,t}-\pi^P_{H,t-1}-\pi^{P*}_{F,t}+\pi^{P*}_{F,t-1}+\pi^*_{H,t}-\pi_{F,t}-2m_t+2m_{t-1}\\
+e_t-e_{t-1}
\end{array}\right)^2\\
-(1-\chi)\left(\begin{array}{l}
\chi(1-\chi)(1-\theta)\left(\pi^{L,o*}_{Ht}-\pi^{P,o}_{Ht}-\pi^{L,o}_{Ft}+\pi^{P,o*}_{Ft}+2m_t\right)\\
+\pi^P_{H,t}-\pi^P_{H,t-1}-\pi^{P*}_{F,t}+\pi^{P*}_{F,t-1}+\pi^*_{H,t}-\pi_{F,t}-2m_t+2m_{t-1}
\end{array}\right)^2\\
+\tfrac{(1-\chi)}{\theta}\left[(1-\theta)\chi\left(\pi^{L,o}_{Ht}-\pi^{P,o}_{Ht}-\pi^{L,o*}_{Ft}+\pi^{P,o*}_{Ft}\right)+m_t-\theta m_{t-1}\right]^2\\
-\tfrac{(1-\chi)}{\theta}\left[(1-\theta)\chi\left(\pi^{L,o}_{Ht}-\pi^{P,o}_{Ht}-\pi^{L,o*}_{Ft}+\pi^{P,o*}_{Ft}\right)\right]^2
\end{array}\right)
\end{array}\right],
\end{aligned}
\tag{31}
$$

where $\delta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Here $\pi^{L,o}_{Ht}$, $\pi^{L,o*}_{Ht}$ and $\pi^{P,o}_{Ht}$ denote the optimal price for Home sales of LCP Home products, that for Foreign sales of LCP Home products, and that for Home and Foreign sales of PCP Home products, respectively and all prices are deflated by PPI. $\pi^{L,o*}_{Ft}$, $\pi^{L,o}_{Ft}$ and $\pi^{P,o*}_{Ft}$ are their Foreign counterparts.[38] The relation (31) clearly shows the effect of dual-currency invoicing on the objective of cooperative central banks. By the inspection of the first four rows in (31), we can observe that the weighted sum of cross-section variances of consumer prices includes not only squared PPI inflation rates, but also the quadratic differences between prices of LCP and PCP products which are newly quoted at time $t$.[39] Note that LCP firms segment markets by country

[37] For full derivations, see the online appendix G.4: Price Dispersion of PCP and LCP Products.

[38] Formally, these reoptimized prices of LCP and PCP products are defined as

$$
\pi^{L,o}_{Ht}=\log\left(\frac{P^{L,o}_{Ht}}{P_{Ht}}\right),\qquad \pi^{L,o*}_{Ht}=\log\left(\frac{P^{L,o*}_{Ht}}{P^*_{Ht}}\right),\qquad \pi^{P,o}_{Ht}=\log\left(\frac{P^{P,o}_{Ht}}{P_{Ht}}\right),
$$
$$
\pi^{L,o*}_{Ft}=\log\left(\frac{P^{L,o*}_{Ft}}{P^*_{Ft}}\right),\qquad \pi^{L,o}_{Ft}=\log\left(\frac{P^{L,o}_{Ft}}{P_{Ft}}\right),\qquad \pi^{P,o*}_{Ft}=\log\left(\frac{P^{P,o*}_{Ft}}{P^*_{Ft}}\right).
$$

[39] In the first squared terms in the third and fourth rows of (31), currency misalignment $m_t$ is added only for the currency adjustment between Home and Foreign prices of a same good. Recall $\pi^{L,o*}_{Ht}=p^{L,o*}_{Ht}-p^*_{Ht}$, $\pi^{L,o}_{Ft}=p^{L,o}_{Ft}-p_{Ft}$ and $m_t=e_t+p^*_{Ht}-p_{Ht}=e_t+p^*_{Ft}-p_{Ft}$. For example, $\pi^{L,o*}_{Ht}+m_t$ is the optimal price for Foreign sales of LCP

and set prices in the importer's currency while PCP firms set one single price for all markets in the exporter's currency. If PCP and LCP products from the same source country are sold at different prices, this creates additional cross-sectional price dispersion, which leads to output(labor) misallocation given that they share the same markups and marginal costs. If we restrict our focus on symmetric equilibrium, Home(Foreign) firms' reoptimized prices in Home(Foreign) markets are equalized across LCP and PCP products: $\pi_{Ht}^{L,o} = \pi_{Ht}^{P,o}$ and $\pi_{Ft}^{L,o*} = \pi_{Ft}^{P,o*}$. Thus the variances of consumer prices only contain price dispersion of LCP and PCP products in export markets:

$$
\begin{aligned}
&\left(\tfrac{\nu}{2}\right)\sigma_{P_H,t}{}^2 + \left(1-\tfrac{\nu}{2}\right)\sigma_{P_H^*,t}{}^2 + \left(\tfrac{\nu}{2}\right)\sigma_{P_F^*,t}{}^2 + \left(1-\tfrac{\nu}{2}\right)\sigma_{P_F,t}{}^2 \\
= \quad & \left(\tfrac{1}{\delta}\right)\left[\begin{array}{ll}
 & +\left(\tfrac{\nu}{2}\right)(\pi_{H,t})^2 + \left(1-\tfrac{\nu}{2}\right)(\pi_{H,t}^*)^2 + \left(\tfrac{\nu}{2}\right)(\pi_{F,t}^*)^2 + \left(1-\tfrac{\nu}{2}\right)(\pi_{F,t})^2 \\
+\left(1-\tfrac{\nu}{2}\right) & \tfrac{\chi(1-\chi)^2(1-\theta)^2}{\theta}\left(\left[\pi_{Ht}^{L,o*} - \pi_{Ht}^{P,o} + m_t\right]^2 + \left[\pi_{Ft}^{L,o} - \pi_{Ft}^{P,o*} - m_t\right]^2\right) \\
+\left(1-\tfrac{\nu}{2}\right) & \left(\begin{array}{l}
+(1-\chi)\left[1+2\theta\chi(1-\chi)\right](e_t - e_{t-1})^2 + \frac{2(1-\chi)}{\theta}(m_t - \theta m_{t-1})^2 \\
+(1-\chi)\left(\begin{array}{l}
\chi(1-\chi)(1-\theta)\left(\pi_{Ht}^{L,o*} - \pi_{Ht}^{P,o} - \pi_{Ft}^{L,o} + \pi_{Ft}^{P,o*} + 2m_t\right) \\
+\pi_{H,t}^{P} - \pi_{H,t-1}^{P} - \pi_{F,t}^{P*} + \pi_{F,t-1}^{P*} + \pi_{H,t}^* - \pi_{F,t} \\
-2m_t + 2m_{t-1} + e_t - e_{t-1}
\end{array}\right)^2 \\
-(1-\chi)\left(\begin{array}{l}
\chi(1-\chi)(1-\theta)\left(\pi_{Ht}^{L,o*} - \pi_{Ht}^{P,o} - \pi_{Ft}^{L,o} + \pi_{Ft}^{P,o*} + 2m_t\right) \\
+\pi_{H,t}^{P} - \pi_{H,t-1}^{P} - \pi_{F,t}^{P*} + \pi_{F,t-1}^{P*} + \pi_{H,t}^* - \pi_{F,t} \\
-2m_t + 2m_{t-1}
\end{array}\right)^2
\end{array}\right)
\end{array}\right].
\end{aligned} \tag{32}
$$

The second row in (32) indicates that currency misalignment disperse prices of LCP and PCP products imported from the same country even if they are newly priced at time $t$.

Lastly, exchange rate movements also worsen the discrepancy between prices of LCP and PCP goods which have not been reoptimized. Observe additional quadratic terms interacting with nominal exchange rate growth and currency misalignment in the last row of equations (31) and (32). Under full PCP ($\chi = 0$), the law of one price implies that exchange rate movements do not play any role in price dispersion. Under full LCP ($\chi = 1$), exchange rate changes only affect the cross-country difference of Home and Foreign prices of identical goods through sluggish adjustments in the relative price of imports($s_t$). But nominal exchange rate movements do not further exacerbate price dispersion among Home(Foreign) goods sold in Foreign(Home) markets.

Home products which are evaluated in Home currency and deflated by Home PPI, $p_{Ht}$.

By contrast, if LCP and PCP products coexist in export markets and their price adjustments are sluggish in invoicing currencies, there are dynamic effects of exchange rate movements on imported-goods price dispersion. For illustration, recall Calvo pricing friction in which firms reset prices with a probability $1-\theta$. In Home markets at time $t$, there are mass $\chi(1-\theta)\theta^2$ of Foreign LCP products sold at a price $P^{L,o}_{F,t-2}$ and mass $\chi(1-\theta)\theta$ of Foreign LCP products with a price $P^{L,o}_{F,t-1}$. At the same time, there are mass $(1-\chi)(1-\theta)\theta^2$ of Foreign PCP products sold at $\mathcal{E}_t P^{P,o*}_{F,t-2}$ and mass $(1-\chi)(1-\theta)\theta$ of Foreign PCP products with $\mathcal{E}_t P^{P,o*}_{F,t-1}$. It is obvious that only the consumer prices of PCP products respond to the nominal exchange rate $\mathcal{E}_t$ and this leads to the cumulative discrepancy between prices of LCP and PCP goods which have not been adjusted by their producers. In addition, those Foreign PCP products were priced at $\mathcal{E}_{t-1} P^{P,o*}_{F,t-2}$ and $\mathcal{E}_{t-1} P^{P,o*}_{F,t-1}$ in the previous period. Since price dispersion a period ago carries over into the current period, exchange rate growth cumulatively impinges on price dispersion between LCP and PCP goods in export markets.

In a nutshell, exchange rate movements change consumer prices of PCP exported goods even if producers do not newly update their prices. Since local prices of a fraction $\theta$ of LCP exported goods are completely insulated from exchange rate changes, this discrepancy between prices of PCP and LCP goods leads to additional relative price distortion in export markets. The effect of exchange rate movements on price distortion within the group of imported goods is absent in the two polar PCP and LCP regimes. Therefore, this explains the presence of quadratic terms interacting with nominal exchange rate growth and currency misalignment in (31) and (32). In the appendix, we provide the quantitative importance of output gap, external targets, and price dispersion in the loss function based upon the parametrization given in Table 2.

### 4.2 Optimal Monetary Policy under Pricing-to-Market and Imperfect Financial Market Integration

In order to complete the optimization problem for central banks, we need pricing equations of PCP firms in Home and Foreign given by[40]

$$\frac{\pi^P_{Ht}-\theta\pi^P_{H,t-1}+\theta\pi_{Ht}}{1-\theta} = (1-\beta\theta)\begin{bmatrix} \left(\frac{\sigma}{D}+\phi\right)\widetilde{y}^R_t + (\sigma+\phi)\widetilde{y}^W_t \\ + \quad \frac{D-\nu+1}{2D}(m_t+f_t)+\frac{1}{\delta}u_t \end{bmatrix} + \frac{\beta\theta}{1-\theta}\mathbb{E}_t\left[\pi^P_{H,t+1}-\theta\pi^P_{H,t}+\pi_{H,t+1}\right],$$

$$\frac{\pi^{P*}_{Ft}-\theta\pi^{P*}_{F,t-1}+\theta\pi^*_{Ft}}{1-\theta} = (1-\beta\theta)\begin{bmatrix} \left(-\frac{\sigma}{D}-\phi\right)\widetilde{y}^R_t + (\sigma+\phi)\widetilde{y}^W_t \\ + \quad \frac{-D+\nu-1}{2D}(m_t+f_t)+\frac{1}{\delta}u^*_t \end{bmatrix} + \frac{\beta\theta}{1-\theta}\mathbb{E}_t\left[\pi^{P*}_{F,t+1}-\theta\pi^{P*}_{F,t}+\pi^*_{F,t+1}\right]. \tag{33}$$

Here PPI inflation, $\pi_{Ht}$ and $\pi^*_{Ft}$, can be replaced with cross-country CPI inflation rates and the relative price of Foreign goods through the equations[41]:

$$\pi_{Ht} = \pi^W_t + \pi^R_t - \left(1-\frac{\nu}{2}\right)\Delta s_t \quad \text{and} \quad \pi^*_{Ft} = \pi^W_t - \pi^R_t + \left(1-\frac{\nu}{2}\right)\Delta s_t.$$

Therefore, the optimal cooperative monetary policy chooses a path for output gaps($\widetilde{y}^R_t, \widetilde{y}^W_t$), inflation rates($\pi^R_t, \pi^W_t$), currency misalignment($m_t$), demand imbalance($f_t$), and seven relative price terms($s_t, \pi^L_{Ht}, \pi^{L*}_{Ht}, \pi^P_{Ht}, \pi^{L*}_{Ft}, \pi^L_{Ft}, \pi^P_{Ft}$) to minimize the expected present value of the global welfare loss (27) given by $\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t L_t$, subject to the equilibrium condition stemming from financial market structure (19), aggregate demand relation (20), three aggregate supply equations (24, 25, 26), four relations for PPI-deflated price indexes of LCP and PCP products (28), and two pricing equations of PCP firms (33). Due to its complexity, analytically tractable targeting rules are not available under generic $\lambda \in [0,1]$ and $\chi \in [0,1]$ and thus we will conduct the numerical analysis on optimal monetary policy in the subsequent section 5. However, we can restrict our focus on specific parametrization to find out analytical targeting rules under the two polar PCP($\chi = 0$) and LCP($\chi = 1$) regimes. In section 4.3, we assume the consumption utility is in log($\sigma = 1$) and the household's labor supply is perfectly elastic($\phi = 0$) to characterize analytical targeting rules.

[40] See equations (170) and (171) in the online appendix H for derivations.

[41] For the derivation, refer to the online appendix F.4: The Relationship between CPI and PPI Inflation Rates.

### 4.3 Simple Targeting Rules and Analytical Results

The goal of our analytical analysis is to investigate the effect of imperfect risk sharing on optimal targeting rules under PCP and LCP. In the PCP regime($\chi = 0$) under imperfect financial integration($\lambda \in [0,1]$), we can derive closed-form targeting rules without any parameter restrictions. By contrast, in the LCP regime($\chi = 1$), closed-form targeting rules are available under the log consumption utility($\sigma = 1$) and the infinite Frisch elasticity of labor supply($\phi = 0$). For the comparison between PCP and LCP regimes, we assume $\sigma = 1$ and $\phi = 0$ throughout this subsection.

Under PCP ($\chi = 0$), linear target criteria[42] can be seen to be

$$0 = \xi\left(\tfrac{\pi_{Ht}+\pi^*_{Ft}}{2}\right) + \left(\widetilde{y}^W_t - \widetilde{y}^W_{t-1}\right), \tag{34}$$

$$0 = \xi\left(\tfrac{\pi_{Ht}-\pi^*_{Ft}}{2}\right) + \left(\tfrac{2}{[2+\Xi_1(D-\nu+1)]}\right)\left(\widetilde{y}^R_t - \widetilde{y}^R_{t-1}\right) + \left(\tfrac{\Xi_1\epsilon\nu(2-\nu)}{2[2+\Xi_1(D-\nu+1)]}\right)(f_t - f_{t-1}), \tag{35}$$

where $D \equiv 1+\nu(2-\nu)(\epsilon-1) = (\nu-1)^2+\epsilon\nu(2-\nu)$ and $\Xi_1 \equiv \frac{2\lambda(2-\nu)\nu(\epsilon-1)}{\lambda(2-\nu)(\epsilon\nu-\nu+1)+2(1-\lambda)D}$.

Under LCP ($\chi = 1$), relatively simple expression for the targeting rules[43] emerges as follows:

$$0 = \xi\left(\tfrac{\pi_t+\pi^*_t}{2}\right) + \left(\widetilde{y}^W_t - \widetilde{y}^W_{t-1}\right), \tag{36}$$

$$0 = \xi\left(\tfrac{\pi_t-\pi^*_t}{2}\right) + \left(\tfrac{\nu-1}{D}\right)\left(\widetilde{y}^R_t - \widetilde{y}^R_{t-1}\right) + \left(\tfrac{\epsilon\nu(2-\nu)}{2D}\right)(m_t + f_t - m_{t-1} - f_{t-1}). \tag{37}$$

As is well known from Engel (2011), optimal monetary policy under PCP targets PPI inflation, $\pi_{Ht}$ and $\pi^*_{Ft}$, while the monetary policy under LCP targets CPI inflation, $\pi_t$ and $\pi^*_t$. CPI and PPI inflation rates in cross-country difference are related through the equilibrium condition[44] given by

$$\frac{\pi_t - \pi^*_t}{2} = \frac{\pi_{Ht} - \pi^*_{Ft}}{2} + \left(1 - \frac{\nu}{2}\right)\Delta s_t. \tag{38}$$

Since the relative price growth of Foreign goods ($\Delta s_t$) in Home fluctuates in response to shocks, CPI inflation stabilization does not lead to PPI inflation stabilization simultaneously and vice versa.

Observe that targeting rules (34) and (36) have the common relation for the cross-country sum of inflation rates since the equality, $\frac{\pi_{Ht}+\pi^*_{Ft}}{2} = \frac{\pi_t+\pi^*_t}{2}$, holds due to zero export premium ($z_t = 0$).

[42]See the online appendix I for derivation. This targeting rule corresponds to equation (38) in Engel (2014).

[43]The derivation is given in the online appendix J.1.

[44]The relation (38) holds without any parameter restrictions and regardless of the degree of exchange rate pass-through ($\chi \in [0,1]$). The derivation is shown in the online appendix F.4.

In response to the world output gap growth, the global planner reduces the cross-country sum of inflation with the weight of the inverse of the substitutability among varieties ($\xi > 1$) as discussed in the previous literature.[45] On the other hand, the cross-country difference rules (35) and (37) imply that the global planner adjusts relative inflation in accordance with the growth of relative output gap and the growth of external imbalance, i.e., currency misalignment ($m_t$) plus demand imbalance ($f_t$), where currency misalignment is zero under PCP. The crucial difference of optimal monetary policy between PCP and LCP regimes is that the monetary authority cannot manage the relative price of Foreign goods ($s_t$) and demand imbalance ($f_t$) under LCP while the authority can directly affect $s_t$ and $f_t$ by policy under PCP.[46]

**Proposition 3.** *Suppose the consumption utility is in log ($\sigma = 1$) and the household's labor supply is perfectly elastic ($\phi = 0$). Under imperfect financial integration across countries ($\lambda \in [0, 1]$), the relative price of imported goods ($s_t$) and demand imbalance ($f_t$) are independent of policy in LCP while they are dependent on policy in PCP.*

Another interesting observation is that the degree of international financial integration ($\lambda$) appears only in the PCP targeting rule (35), not in the LCP targeting rule (37). The cross-country targeting rule (37) under LCP always reduces relative CPI inflation in response to the relative output gap growth and/or the growth of external imbalance regardless of the degree of imperfect financial integration.[47] In the PCP cross-country targeting rule (35), the direction of the response of relative PPI inflation with respect to a change in $\lambda$ is determined by the elasticity of substitution between Home and Foreign consumption baskets ($\epsilon$). Suppose the cross-country trade elasticity is greater than unity ($\epsilon > 1$). Then the global planner under PCP reduces relative PPI inflation in accordance with the rise in relative output gap growth and/or demand imbalance growth.[48] The PCP targeting rule (35) further implies that when the economy approaches financial autarky ($\lambda \to 1$) and thus cross-country risk sharing gets worse, relative PPI inflation should respond more to demand imbalance and less to relative output gap.[49] We provide proposition 4 to highlight this

[45] See Corsetti et al. (2010), Engel (2011), Engel (2014) and Corsetti et al. (2020) among others.

[46] Sections I.2 and J.2 in the online appendix establish the dependence of $s_t$ and $f_t$ on policy under PCP and their independence from policy under LCP, respectively. Note from Corsetti et al. (2020) that these also hold true under the economy with uncontingent bonds.

[47] The irrelevance of the degree of cross-country risk sharing for optimal targeting rules under LCP was also discussed in Engel (2014) and Corsetti et al. (2020).

[48] It is crucial to observe that the cross-country trade elasticity governs not only the direction of the change in $\Xi_1 \equiv \frac{2\lambda(2-\nu)\nu(\epsilon-1)}{\lambda(2-\nu)(\epsilon\nu-\nu+1)+2(1-\lambda)D}$ with respect to $\lambda$: $\frac{\partial \Xi_1}{\partial \lambda} > 0$ for $\epsilon > 1$ and $\frac{\partial \Xi_1}{\partial \lambda} < 0$ for $\epsilon < 1$, but also its sign: $\Xi_1 > 0$ for $\epsilon > 1$ and $\Xi_1 < 0$ for $\epsilon < 1$, where $\frac{\partial \Xi_1}{\partial \lambda} = \frac{4(2-\nu)\nu(\epsilon-1)D}{[\lambda(2-\nu)(\epsilon\nu-\nu+1)+2(1-\lambda)D]^2}$.

[49] To see this, note that $\frac{\partial \Xi_1}{\partial \lambda} > 0$ when $\epsilon > 1$. Under $\epsilon > 1$, the coefficient for relative output gap in (35) is

result.

**Proposition 4.** *Under producer-currency pricing with the trade elasticity greater than unity, optimal targeting rules imply that the cross-country PPI inflation difference should respond less to relative output gap growth and more to demand imbalance growth as cross-country risk sharing gets worse. The PPI inflation responses are in the same direction: the Home PPI inflation should decrease according to rises in relative output gap and/or demand imbalance, holding the Foreign PPI inflation constant.*

By contrast, if the trade elasticity is less than unity ($\epsilon < 1$), the cross-country targeting rule (35) under PCP implies that relative PPI inflation responds stronger to both relative output gap growth and demand imbalance growth as cross-country risk sharing deteriorates ($\lambda \to 1$), but signs of the responses are in the opposite. The monetary authority contracts relative PPI inflation in response to the increase in relative output gap growth whereas she raises relative PPI inflation according to the rise in demand imbalance growth.[50] Proposition 5 summarizes this point.

**Proposition 5.** *Under producer-currency pricing with the trade elasticity less than unity, optimal targeting rules imply that the cross-country PPI inflation difference should respond more aggressively to both relative output gap growth and demand imbalance growth as cross-country risk sharing gets worse. The PPI inflation responses are in the opposite direction: the Home PPI inflation should fall according to the rise in relative output gap whereas it should increase with higher demand imbalance, holding the Foreign PPI inflation constant.*

Lastly, it is important to note that under the Cole and Obstfeld specification: $\sigma = \epsilon = 1$ and $\phi = 0$, optimal monetary policy responses are identical between our economy with capital controls and the economy with uncontingent bonds (Corsetti et al. (2020)).[51] With the unitary trade

positive and gets smaller by the increase in $\lambda$: $\frac{\partial\left(\frac{2}{\xi[2+\Xi_1(D-\nu+1)]}\right)}{\partial\lambda} = \left(\frac{-2(D-\nu+1)}{\xi[2+\Xi_1(D-\nu+1)]^2}\right)\frac{\partial\Xi_1}{\partial\lambda} < 0$. Here $\epsilon > 1$ implies $\epsilon\nu - \nu + 1 > 0$ and thus we use $D - \nu + 1 > 0$. In addition, the coefficient for the demand imbalance in (35) is positive and gets larger as $\lambda$ ascends: $\frac{\partial\left(\frac{\Xi_1\epsilon\nu(2-\nu)}{2\xi[2+\Xi_1(D-\nu+1)]}\right)}{\partial\lambda} = \left(\frac{\epsilon\nu(2-\nu)}{\xi[2+\Xi_1(D-\nu+1)]^2}\right)\frac{\partial\Xi_1}{\partial\lambda} > 0$.

[50] To see this, note that $\frac{\partial\Xi_1}{\partial\lambda} < 0$ if $\epsilon < 1$. Under $\epsilon < 1$, the coefficient for relative output gap in (35) is still positive and it increases with respect to the rise in $\lambda$: $\frac{\partial\left(\frac{2}{\xi[2+\Xi_1(D-\nu+1)]}\right)}{\partial\lambda} = \left(\frac{-2(D-\nu+1)}{\xi[2+\Xi_1(D-\nu+1)]^2}\right)\frac{\partial\Xi_1}{\partial\lambda} > 0$. Here we need further restriction by $\frac{\nu-1}{\nu} < \epsilon < 1$. There is no well-defined equilibrium under $\epsilon < \frac{\nu-1}{\nu}$. On the other hand, under $\epsilon < 1$, the coefficient for the demand imbalance in (35) is negative and it decreases according to the increase in $\lambda$: $\frac{\partial\left(\frac{\Xi_1\epsilon\nu(2-\nu)}{2\xi[2+\Xi_1(D-\nu+1)]}\right)}{\partial\lambda} = \left(\frac{\epsilon\nu(2-\nu)}{\xi[2+\Xi_1(D-\nu+1)]^2}\right)\frac{\partial\Xi_1}{\partial\lambda} < 0$, that is, it becomes larger negative number.

[51] See the equivalence results presented in online appendices I.4 for PCP and J.4 for LCP.

elasticity, the cross-country targeting rules in (35) and (37) reduce to

$$\begin{aligned}
&[PCP] \quad 0 = \xi \left(\tfrac{\pi_{Ht} - \pi^*_{Ft}}{2}\right) + \left(\widetilde{y}^R_t - \widetilde{y}^R_{t-1}\right), \\
&[LCP] \quad 0 = \xi \left(\tfrac{\pi_t - \pi^*_t}{2}\right) + (\nu - 1)\left(\widetilde{y}^R_t - \widetilde{y}^R_{t-1}\right) + \left(\tfrac{\nu(2-\nu)}{2}\right)(m_t + f_t - m_{t-1} - f_{t-1}) .
\end{aligned} \tag{39}$$

The cross-country targeting rules (39) exactly match those analyzed in Corsetti et al. (2020). Now the degree of financial market imperfection ($\lambda$) does not play any role in optimal targeting rules and the demand imbalance in (19) becomes independent of policy under both PCP and LCP regimes: $f_t = \left(\frac{-\lambda(2-\nu)}{2-\lambda\nu}\right)\left(\zeta_{c,t} - \zeta^*_{c,t}\right)$.[52] In turn, net exports are simply derived by $nx_t = -f_t - \zeta_{c,t} + \zeta^*_{c,t}$, implying that net exports and net capital flows are also exogenous to monetary policy.[53] Therefore in response to inefficient capital flows, optimal policy responses under the uncontingent-bond economy discussed in Corsetti et al. (2020) carry over to our economy with state-contingent bonds and capital controls under the Cole and Obstfeld specification: $\sigma = \epsilon = 1$ and $\phi = 0$. One notable difference is that under the economy with uncontingent bonds, demand imbalance follows a random walk and so do net exports and net capital flows. But in our economy, they are stationary and become more volatile in response to the cross-country difference of preference shocks when the economy approaches financial autarky.[54]

## 5 Numerical Analysis

In this section, we extend our analysis to the general case where there is heterogeneity among exporters in invoicing currencies captured by the parameter $\chi$ ranging from zero (PCP) to one (LCP). We also analyze the effect of imperfect financial market integration by varying the parameter $\lambda$ from zero (perfect risk sharing) to one (financial autarky). Since analytical allocations are not available in this general specification, we turn to the numerical analysis to examine interactions between incomplete exchange rate pass-through and imperfect financial market integration in the conduct of optimal monetary policy.

The parameters are calibrated as in Table 2. The discount rate and the probability of resetting

[52] The relative price of Foreign goods ($s_t$) is still dependent on policy under PCP even with the Cole and Obstfeld specification.

[53] Note that net exports are related to net capital outflow through the balance of payments. For details, refer to the online appendix F.1.

[54] To see this, observe that the coefficient $\left(\frac{-\lambda(2-\nu)}{2-\lambda\nu}\right)$ in $f_t$ becomes a larger negative number as $\lambda$ approaches one ($\lambda \to 1$): $\frac{\partial\left(\frac{-\lambda(2-\nu)}{2-\lambda\nu}\right)}{\partial\lambda} = \left(\frac{-2(2-\nu)}{(2-\lambda\nu)^2}\right) < 0$.

Table 2: Parametrization

| Parameter | Description | Value | Source |
|---|---|---|---|
| $\beta$ | The discount rate | 0.99 | 4% annual rate of return |
| $\sigma$ | Relative risk aversion | 2.00 | Senay and Sutherland (2019) |
| $\kappa$ | Weight on labor disutility | 1.00 | Engel (2011) |
| $\phi$ | Inverse Frisch labor-supply elasticity | 2.00 | Senay and Sutherland (2019) |
| $\nu$ | Consumption home bias | 1.50 | Engel (2011) |
| $\theta$ | Calvo pricing friction | 0.75 | Price changes in 4 quarters on average |
| $\xi$ | Elasticity of substitution bet. varieties | 4.00 | GMM Micro-Elasticity: Feenstra et al. (2018) |
| $\epsilon$ | Cross-country Trade Elasticity | 2.00 | GMM Macro-Elasticity: Feenstra et al. (2018) |
| $\lambda$ | Degree of financial market integration | $[0, 1]$ | From full risk sharing (0) to financial autarky (1) |
| $\chi$ | Degree of exchange rate pass-through | $[0, 1]$ | From full PCP (0) to full LCP (1) |
| $\rho_A$ | Persistence of Home productivity shock | 0.95 | Senay and Sutherland (2019) |
| $\sigma_A$ | Size of Home productivity shock | 0.006 | Senay and Sutherland (2019) |
| $\rho_{\zeta_C}$ | Persistence of Home preference shock | 0.90 | Senay and Sutherland (2019) |
| $\sigma_{\zeta_C}$ | Size of Home preference shock | 0.01 | Senay and Sutherland (2019) |

prices are set at the conventional values (set to quarterly frequency): $\beta = 0.99$ and $1 - \theta = 0.25$, respectively. As in Engel (2011) and Corsetti et al. (2020), the weight on labor disutility is set to $\kappa = 1$ and the home-bias parameter is fixed at $\nu = 1.5$, which implies households put a weight of 3/4th on domestically produced goods, holding all prices equal. We choose the elasticity of substitution between differentiated varieties to be the value, $\xi = 4.00$, so that the monopoly markup amounts to 33%. This corresponds to the value of the micro-Armington elasticity estimated by GMM using U.S. data from Feenstra et al. (2018). For the cross-country trade elasticity, we follow the rule of two (Feenstra et al. (2018)) in converting the micro-elasticity into the macro-Armington elasticity: $\epsilon = 2$. We set relative risk aversion to $\sigma = 2$ and the inverse Frisch labor-supply elasticity to $\phi = 2$ following Senay and Sutherland (2019) which are consistent with the estimates of Smets and Wouters (2007). We also take parametrization for productivity and preference shock processes from Senay and Sutherland (2019) which are based upon Smets and Wouters (2007) and Corsetti et al. (2010).

Since incomplete exchange rate pass-through ($0 \leq \chi \leq 1$) and imperfect financial market integration ($0 \leq \lambda \leq 1$) create an additional dimension to policy trade-offs deviating from an inward-looking policy, we abstract from a time-varying markup charged by workers. Due to the symmetry between Home and Foreign in equilibrium, we only focus on shocks originating in the Home country. Before providing the formal analysis, we introduce a measure for welfare costs by which we can rank different regimes of economy in terms of welfare. We define *welfare costs*, $\eta^C$, as the fraction of consumption which households in both countries should forgo to be as well off under the efficient equilibrium as under some particular monetary regime, given by

$$\begin{aligned} &\mathbb{E}_0 \sum_{t=0}^{\infty} \left[ \beta^t \left\{ \frac{\zeta_{C,t}(C_t)^{1-\sigma} + \zeta_{C,t}^*(C_t^*)^{1-\sigma}}{1-\sigma} - \kappa \frac{(N_t)^{1+\phi} + (N_t^*)^{1+\phi}}{1+\phi} \right\} \right] \\ = \; &\mathbb{E}_0 \sum_{t=0}^{\infty} \left[ \beta^t \left\{ \frac{\zeta_{C,t}\big((1-\eta^C)\overline{C}_t\big)^{1-\sigma} + \zeta_{C,t}^*\big((1-\eta^C)\overline{C}_t^*\big)^{1-\sigma}}{1-\sigma} - \kappa \frac{\big(\overline{N}_t\big)^{1+\phi} + \big(\overline{N}_t^*\big)^{1+\phi}}{1+\phi} \right\} \right], \end{aligned} \tag{40}$$

where $\overline{C}_t$, $\overline{C}_t^*$, $\overline{N}_t$ and $\overline{N}_t^*$ are consumption and labor in Home and Foreign under the efficient equilibrium.[55]

## 5.1 The Effect of Supply Shocks under Incomplete Exchange Rate Pass-Through and Imperfect Financial Integration

Consider the world economy perturbed by a shock to Home productivity. Figures 4, 5, and 6 summarize its equilibrium results. In Figure 4, charts in the first row display welfare cost under optimal monetary policy relative to efficient outcome. It is worth noting that if risk sharing is perfect ($\omega = 0$) and exchange rate pass-through is complete ($\chi = 0$), allocations under optimal monetary policy incur zero welfare cost relative to efficient equilibrium, meaning that the policy attains the first-best outcome.

Given a certain degree of risk sharing ($\omega = \lambda/(1-\lambda) > 0$), we can observe that the welfare cost monotonically increases as the degree of exchange rate pass-through declines ($\chi \to 1$). On the other hand, when the exchange rate pass-through is incomplete ($\chi > 0$), the policymaker can improve the welfare by restricting cross-country capital flows because capital controls can partially offset distortions from the incomplete ERPT. However, welfare gains of optimal capital controls relative to perfect financial markets are quantitatively small.[56]

[55] For details on efficient allocations, see the online appendix D. For linearized efficient allocations, refer to the online appendix F.6. In computing welfare costs, first we derive a second-order approximation to the world welfare as follows:

$$\begin{aligned} \mathcal{W}(\mathfrak{s}_{t-1}, \mathfrak{z}_t) &= \frac{\zeta_{C,t} C(\mathfrak{s}_{t-1}, \mathfrak{z}_t)^{1-\sigma} + \zeta_{C,t}^* C^*(\mathfrak{s}_{t-1}, \mathfrak{z}_t)^{1-\sigma}}{1-\sigma} - \kappa \frac{N(\mathfrak{s}_{t-1}, \mathfrak{z}_t)^{1+\phi} + N^*(\mathfrak{s}_{t-1}, \mathfrak{z}_t)^{1+\phi}}{1+\phi} + \beta \mathbb{E}_t \left[ \mathcal{W}(\mathfrak{s}_t, \mathfrak{z}_{t+1}) \right] \\ &\approx \overline{\mathcal{W}} + \frac{\Upsilon_{\mathcal{W},0}}{2} + \mathcal{C}\mathfrak{s}_{t-1} + \mathcal{D}\mathfrak{z}_t + \frac{\Upsilon_{\mathcal{W},1}}{2} \left( \mathfrak{s}_{t-1} \otimes \mathfrak{s}_{t-1} \right) + \frac{\Upsilon_{\mathcal{W},2}}{2} \left( \mathfrak{z}_t \otimes \mathfrak{z}_t \right) + \Upsilon_{\mathcal{W},3} \left( \mathfrak{s}_{t-1} \otimes \mathfrak{z}_t \right), \end{aligned}$$

where $\mathfrak{s}_t$ and $\mathfrak{z}_t$ stand for relevant endogenous and exogenous state vectors. $\overline{\mathcal{W}}$ denotes the steady state value of the world welfare and $\{\mathcal{C}, \mathcal{D}, \Upsilon_{\mathcal{W},0}, \Upsilon_{\mathcal{W},1}, \Upsilon_{\mathcal{W},2}, \Upsilon_{\mathcal{W},3}\}$ represent approximation coefficients. Conditioning on the efficient steady state, we compute the conditional welfare by $\mathcal{W}_0 = \overline{\mathcal{W}} + \frac{\Upsilon_{\mathcal{W},0}}{2}$. Then we can derive welfare costs $\eta^C$ by using $\mathcal{W}_0$. The reader can refer to Schmitt-Grohé and Uribe (2004), Schmitt-Grohé and Uribe (2007) and Faia and Monacelli (2007) for the derivation of the second-order approximation to the conditional welfare.

[56] The consumption-equivalent welfare gains under optimal capital controls relative to perfect financial markets are less than 1 basis point in response to productivity shocks.

Three charts in the second row of Figure 4 compare welfare costs among strict CPI inflation targeting (CPIT or CPI TGT), strict PPI inflation targeting (PPIT or PPI TGT) and optimal monetary policy (OMP). The first two charts present the welfare cost difference between CPIT (PPIT) and OMP. Zero value in the vertical axis implies the welfare under CPIT (PPIT) is the same as the welfare under OMP. In the third chart, we plot the region of $\chi$ and $\omega$ showing which of strict PPI and CPI inflation targets incurs lower welfare cost: PPI targeting is better under the green area while CPI targeting is more desirable under the violet area. It is apparent that regardless of the degree of risk sharing, strict CPI inflation targeting achieves allocation closer to optimal policy under the low exchange rate pass-through ($0.7 \leq \chi \leq 1$ with $\lambda \approx 0$), whereas strict PPI inflation targeting is more effective under the high exchange rate pass-through ($0 \leq \chi \leq 0.7$ with $\lambda \approx 0$). Since the welfare loss is highly ascribed to relative price distortions across countries under the low exchange rate pass-through[57], the policymaker should stabilize not only prices of domestically produced goods, but also prices of imported goods by targeting CPI rather than PPI.

In the six charts from the bottom of Figure 4, we plot standard deviations of CPI and PPI inflation rates ($\pi_t$, $\pi_{Ht}$), output gap ($y_t - \overline{y}_t$), and external targets ($m_t + f_t$) and impact responses of demand imbalance ($f_t$) and currency misalignment ($m_t$). In the intermediate range of $\chi$ between full PCP and full LCP ($0 < \chi < 1$), the monetary policy optimally trades off CPI and PPI inflation stabilization for output gap and external target stabilization, so that none of target variables are fully stabilized. Volatilities of both output gap ($y_t - \overline{y}_t$) and external imbalance ($m_t + f_t$) monotonically increase as $\chi$ and $\omega$ increase. Demand imbalance ($f_t$) becomes more volatile as cross-country risk sharing gets worse ($\omega \to \infty$) and currency misalignment ($m_t$) deviates further from its efficient counterpart as more firms segment markets ($\chi \to 1$). Interestingly, given a certain degree of inefficient risk sharing ($\omega > 0$), the impact response of demand imbalance ($f_t$) gets weaker when the economy approaches the full LCP regime ($\chi \to 1$). On the other hand, currency misalignment responds less to productivity shocks if the economy approaches financial autarky ($\omega \to \infty$), holding the degree of exchange rate pass-through fixed ($\chi > 0$).

Figures 5 and 6 present impact responses of macroeconomic variables when the economy is perturbed by an one-standard-deviation shock to Home productivity. The figures plot trajectories of responses of efficient allocation (black line), allocation of optimal monetary policy under PCP sticky prices (blue line) and allocation of optimal monetary policy under LCP sticky prices (red

[57] When the exchange rate pass-through gets lower, the price differences between the same goods sold in different countries become larger so that the global welfare gets worse.

line) on impact with respect to the varying degrees of cross-country risk sharing ($\omega$). Under a regime of high exchange rate pass-through (PCP) and high cross-country risk sharing ($\omega \approx 0$), a positive Home productivity shock leads to excess supply of Home goods in Home and Foreign markets, holding prices unadjusted. Thus, the relative price of Foreign goods ($s_t = p_{Ft} - p_{Ht}$) in Home rises while the relative price of Home goods ($s_t^* = p_{Ht}^* - p_{Ft}^*$) in Foreign falls. Accordingly, households in Home and Foreign switch their expenditure from Foreign goods to Home goods, so that Home output increases whereas Foreign output falls. This explains responses under the efficient equilibrium (black line).

Our quantitative contribution is to examine the effect of imperfect risk sharing under optimal monetary policy. First we assume the complete exchange rate pass-through and increase the degree of imperfect risk sharing ($\omega \to \infty$) to see the impact responses of allocations under PCP sticky prices (the blue circled line). When net exports in Home increase in response to positive Home supply shocks, net capital outflows in Home are suppressed by the Home government's tax on external bond holdings, leading to a positive demand imbalance ($f_t > 0$).[58] As the sensitivity of capital controls to the ratio of net capital outflow to consumption expenditure increases ($\omega \to \infty$), the tax rate rises further. Then Home households are more inclined to consume rather than save, so that the excess supply of Home goods shrinks in Home and Foreign relative to the efficient equilibrium if prices are unadjusted. As a result, in contrast to efficient counterparts, the cross-country demand imbalance ($f_t$) rises; the increase in net exports ($nx_t$) are suppressed; the relative price of Foreign goods in Home ($s_t$) rises less; the real exchange rate ($q_t$) depreciates less.[59] Note that capital controls curb net capital outflows from Home so that net capital inflows into Foreign decrease. This leads to lesser Foreign demand on both Foreign and Home goods in compared to the efficient counterpart. Therefore, Foreign consumption on Foreign and Home goods ($c_{Ft}^*$ and $c_{Ht}^*$) falls relative to the efficient level and the deviation of actual output from efficient output expands

[58] A positive tax on net external savings in Home raises the Home household's marginal utility of Home consumption relative to the Foreign household's through the financial market equilibrium condition (13):

$$\zeta_{C,t} C_t(h)^{-\sigma} (1 + \varrho_t) = \zeta_{C,t}^* C_t^* (h^*)^{-\sigma} \left( \frac{1}{Q_t} \right).$$

Note that this relation corresponds to the marginal utility equalization of Home consumption across countries under the complete asset market. If $\varrho_t$ rises above zero, the consumption gap between Home and Foreign ($c_t - c_t^*$) gets larger and the demand imbalance ($f_t$) becomes positive, deviating from zero (see equations (18) and (19)). Since the Home government's taxation makes external savings more costly, Home households increase current consumption.

[59] Note that under perfect financial markets ($\omega = 0$), optimal monetary policy under PCP sticky prices attains the first-best allocation. Therefore, the nominal exchange rate ($e_t$) depreciates further than its efficient counterpart through the relation $\Delta q_t = \Delta e_t + \pi_t^* - \pi_t$ due to $\pi_t^* < 0$ and $\pi_t > 0$ under PCP sticky prices. The optimal monetary policy offsets cross-country CPI inflation movements by depreciating nominal exchange rate to obtain efficient real exchange rate fluctuations.

so that the global welfare loss rises.

Under LCP sticky prices, the effect of imperfect risk sharing is comparable to that under PCP sticky prices; as risk sharing gets worse ($\omega \to \infty$), the qualitative movements of macroeconomic variables under LCP (red line) are the same with those under PCP (blue line). The notable difference is a positive spillover effect of Home productivity shocks on Foreign output ($y_t^*$). Since prices are sticky in local currency and exchange rate pass-through is incomplete, the relative price of Foreign goods in Home ($s_t$) increases much less under LCP than under PCP. Therefore, in response to positive Home productivity shocks, Home households' income rises and this income effect dominates the terms of trade effect. In turn, Home households raise their demand on both Home and Foreign goods, leading to the rise in Foreign output.[60] Therefore, Home demand on both Home and Foreign goods ($c_{Ht}$ and $c_{Ft}$) increase and this leads to further deviation of global output from its efficient counterpart incurring higher global welfare loss than that under PCP.

## 5.2 The Effect of Demand Shocks under Incomplete Exchange Rate Pass-Through and Imperfect Financial Integration

Figures 7, 8 and 9 show equilibrium results when the global economy is disturbed by a shock to Home preference. By the inspection of Figure 7, we can find several notable differences between equilibrium outcomes in response to productivity and preference shocks.

First, as in the result under productivity shocks, the first chart shows there exists an optimal level of capital controls which minimizes welfare cost under incomplete exchange rate pass-through ($\chi > 0$).[61] But, unlike the case with productivity shocks, when the degree of risk sharing is close to the financial autarky ($\omega \to \infty$), the welfare cost decreases as there are more firms who segment markets ($\chi \to 1$). This implies that under shocks to Home preference which directly impact on cross-country risk sharing through equations (18) and (19), sluggish price adjustments in the consumer's currency can partially offset distortions from inefficient capital flows to improve the global welfare.[62]

Second, the implication for inflation targeting exhibits a sharp contrast. As in Figure 4, the

[60] This income effect is labeled as the risk sharing effect in Clarida et al. (2002). Under LCP, the risk sharing effect is more dominant than the terms of trade effect.

[61] As in equilibrium under productivity shocks, welfare gains are also quantitatively small. The consumption-equivalent welfare gains under optimal capital controls in compared to perfect financial markets are less than 1 basis point in response to preference shocks.

[62] Check the third chart in Figure 19 in the online appendix K to catch this pattern more clearly. The third chart in the first row of Figure 19 shows that in response to Home preference shocks, welfare cost under LCP is lower than that under PCP if the degree of international risk sharing is low ($\omega > 4$ or $\lambda > 0.8$).

first two charts in the second row of Figure 7 present the welfare cost difference between CPIT (PPIT) and OMP. Zero value in the vertical axis means CPI (PPI) inflation targeting attains the welfare under optimal monetary policy. The third chart presents the range for $\chi$ and $\omega$ showing which inflation targets are more desirable in terms of welfare: green for PPI and violet for CPI. If the degree of risk sharing is high ($0 \leq \omega \leq 4$), optimal monetary policy still implements PPI inflation targeting under the high exchange rate pass-through ($0 \leq \chi \leq 0.5 \sim 0.7$) and CPI inflation targeting under the low exchange rate pass-through ($0.5 \sim 0.7 \leq \chi \leq 1$). However, given a degree of incomplete ERPT ($\chi > 0$), if the extent of imperfect financial integration exceeds a certain threshold ($\omega > 13$), PPI inflation targeting always dominates CPI inflation targeting in terms of welfare.[63] Indeed, standard deviation of PPI inflation ($\pi_{Ht}$) under optimal monetary policy is stabilized below 0.01% in almost all parameter values for $\chi$ and $\omega$.[64] By contrast, standard deviation of CPI inflation ($\pi_t$) under optimal monetary policy exceeds 0.01% when the degree of risk sharing is low ($\omega \geq 10$). This holds true even when the economy approaches the LCP regime ($\chi \to 1$) if cross-country capital flows are significantly restricted ($\omega \geq 4$).[65] Hence, optimal monetary policy in response to Home preference shocks puts more weight on PPI inflation stabilization than on CPI inflation stabilization even under the low exchange rate pass-through ($\chi \approx 1$) if cross-country risk sharing is low ($\omega \geq 4$).

From the last two charts in the bottom of Figure 7, another important distinction is that the sign of the impact response of demand imbalance is in the opposite to that under productivity shocks: it becomes negative ($f_t < 0$). In response to positive shocks to Home preference, Home households increase external debts to consume more, leading to a surge in net capital inflow. Under capital controls, the Home government imposes a tax on net capital inflow ($\varrho_t < 0$) and thus demand imbalance becomes negative (see equations (5) and (14)). In addition, the sign of the impact response of currency misalignment ($m_t$) changes from negative to positive as risk sharing gets worse under incomplete exchange rate pass-through ($\chi > 0$). A positive response of currency misalignment is desirable in that it counteracts the volatility of demand imbalance. Since

[63] In the range of $\chi$ between 0 and 0.5, there is an area where CPI inflation targeting is more desirable than PPI inflation targeting in the intermediate range of $\omega$ between 4 and 13. However the welfare difference between CPI and PPI inflation targets in that area is less than 0.1 basis point so that the two inflation targeting regimes are indistinguishable in terms of welfare. On the other hand, in those regions of interest where either $0 \leq \omega \leq 4$ or $\omega \geq 13$ with $\chi \approx 1$, the welfare difference between CPI and PPI inflation targets is close to 1 basis point. Hence, we can ignore the lower triangular violet area in the range of $\omega$ between 4 and 13 since CPI and PPI inflation targets do not make any significant difference in welfare.

[64] See the volatility of PPI inflation from the second chart in the third row of Figure 7. The only exception is the region where $\chi$ is nearby one and $\omega$ is around zero.

[65] See the volatility of CPI inflation from the first chart in the third row of Figure 7.

the policymaker targets the sum of demand imbalance and currency misalignment, the opposite responses between currency misalignment and demand imbalance stabilizes the external target $(m_t + f_t)$.

Figures 8 and 9 show impact responses of macroeconomic variables with respect to an one-standard-deviation shock to Home preference. As in section 5.1, we compare efficient allocation (black line) and allocations of optimal monetary policy under PCP and LCP sticky prices (blue and red lines, respectively) by varying the degree of cross-country risk sharing ($\omega$). A positive Home preference shock raises Home demand for Home and Foreign goods if prices are unadjusted. Due to the presence of home bias, higher aggregate demand in Home leads to excess demand for Home goods relative to Foreign goods. Accordingly, the relative price of Foreign goods ($s_t = p_{Ft} - p_{Ht}$) in Home falls and the Home real exchange rate appreciates. Home households raise their external debts to obtain more current consumption, which causes an increase in net capital outflow in Foreign. The rise in external savings in Foreign depresses Foreign households' demand for Home and Foreign goods and therefore Foreign consumption ($c_t^*$) falls while Home consumption ($c_t$) rises. Since the increase in Home demand outweighs the drop in Foreign demand, total output ($y_t$, $y_t^*$) in both countries expands. This explains impact responses under efficient equilibrium (black line).

When the sensitivity of capital controls gets stronger ($\omega \to \infty$) under PCP (blue line), the Home government imposes higher taxes on net capital inflow to reduce trade deficits and the degree of cross-country risk sharing deteriorates. As Home imports are restrained by severer capital controls, it mitigates the excess demand for Home goods relative to Foreign goods induced by home bias and eventually it turns into the excess demand for Foreign goods relative to Home goods if prices are unadjusted.[66] Hence, the relative price of Foreign goods ($s_t$) increases to change its sign from negative to positive after the sensitivity of capital controls exceeds a certain threshold around $\omega = 5$. The decrease in the relative price of Home goods in Foreign ($s_t^*$) leads to higher consumption for Home goods in Foreign ($c_{Ht}^*$) relative to its efficient level.[67] At the same time, the decrease in Foreign consumption on Foreign goods ($c_{Ft}^*$) is suppressed due to falls in Foreign households' external savings resulting from tighter capital controls. As a result, Home output ($y_t$) increases more while Foreign output ($y_t^*$) rises less in compared to efficient counterparts. Allocations under

[66]Recall severer capital controls force the trade to be more balanced, leading the economy to financial autarky in the limiting case. In response to a positive Home preference shock, capital controls suppress rises in net imports in Home. Therefore, stronger capital controls in Home lead to higher excess demand for Foreign goods relative to Home goods and the relative price of Foreign goods ($s_t$) in Home increases.

[67]Recall the equilibrium condition, $s_t^* = -s_t$. The increase in the relative price of Foreign goods in Home ($s_t$) corresponds to the decrease in the relative price of Home goods in Foreign ($s_t^*$).

LCP sticky prices exhibit the same pattern with respect to a change in the degree of risk sharing. The main difference of impact responses under LCP is the dampened adjustment of the relative price of Foreign goods ($s_t$) due to local-currency price stickiness.[68]

# 6 Conclusion

This paper studies optimal cooperative monetary policy under varying degrees of exchange rate pass-through and cross-country financial market integration. We have shown that if countries import goods invoiced in different currencies and their prices are sticky in the currency of invoicing, exchange rate fluctuations disperse consumer prices of imported goods because those goods have different degrees of exchange rate pass-through according to the currency which they are priced in. Hence, the welfare loss from total price dispersion includes additional costs other than PPI or CPI inflation rates. We analytically examine the effect of imperfect financial integration on optimal targeting rules in the knife-edge cases of PCP and LCP. Our numerical analysis also allows us to explicitly take into account the interplay of distortions from sticky prices in multiple invoicing currencies and imperfect risk sharing to compare strict inflation targeting with optimal monetary policy.

Although this paper addresses the role of different degrees of imperfect financial integration combined with incomplete exchange rate pass-through in the two-country open economy, recent literature documents that the vast majority of international trade is invoiced in a few vehicle currencies, dollars and euros dominantly, which are not often national currencies of any trading partners (Goldberg and Tille (2008), Gopinath (2015), Boz et al. (2020)) and there has been much progress in positive consequences of dominant currency pricing (Goldberg and Tille (2016), Amiti et al. (2018), Mukhin (2018), Gopinath et al. (2020)). Under our symmetric two-country world, we can analyze only the two pricing schemes of PCP and LCP for optimal monetary policy cooperation and the data indicate that these must correspond to euros and dollars for the euro area (Figure 1). However the main trading partner of the euro area is not necessarily the U.S. economy. According

[68] Note that in response to positive Home preference shocks, Home households consume more ($c_t \Uparrow$) while Foreign households save more ($c_t^* \Downarrow$). Because of home bias, this leads to excess demand for Home goods relative to Foreign goods in Home and excess supply for Foreign goods relative to Home goods in Foreign if prices are unadjusted. Under perfect risk sharing ($\omega = 0$) with LCP sticky prices, the relative price of Foreign goods in Home ($s_t = p_{Ft} - p_{Ht}$) falls less and that of Home goods in Foreign ($s_t^* = p_{Ht}^* - p_{Ft}^*$) rises less in compared to efficient outcome. Therefore, relative to efficient counterparts, Home consumption on Foreign goods $\left(c_{Ft} = c_t + \epsilon\left(\frac{\nu}{2}\right)(-s_t)\right)$ rises less and Home consumption on Home goods $\left(c_{Ht} = c_t + \epsilon\left(1 - \frac{\nu}{2}\right)(s_t)\right)$ increases more. By contrast, Foreign consumption on Home goods $\left(c_{Ht}^* = c_t^* + \epsilon\left(\frac{\nu}{2}\right)(-s_t^*)\right)$ falls less and Foreign consumption on Foreign goods $\left(c_{Ft}^* = c_t^* + \epsilon\left(1 - \frac{\nu}{2}\right)(s_t^*)\right)$ decreases further.

to Gopinath (2015), significant portion of exporters in the eurozone are euro pricers when they sell to the U.S. market[69], but around 95% of imports and exports in the U.S. are invoiced in dollars (see Figure 14 in the online appendix). Therefore the majority of trading partners of the eurozone other than the U.S. are either euro-currency pricers (LCP) or dollar-currency pricers (neither PCP nor LCP) and it is an important future research to extend our framework to the open economy of at least three countries to examine the implication of dominant-currency pricing in international policy coordination.

Still our paper sheds light on the issue of policy targets under dominant-currency paradigm where some countries use the third currency for trade invoicing. Consider an open economy of three countries, say, the U.S., the eurozone and China. If U.S. firms export to the euro area in dollar pricing (PCP) and Chinese firms export in euro pricing (LCP), the USD/EUR exchange rate movements change euro-currency prices of U.S. goods one-for-one while euro-currency prices of Chinese goods do not respond to exchange rate fluctuations as much in the eurozone markets. Our finding applies to this extended environment and it suggests policymakers should concern exchange rate misalignments not only because of the violation of the law of one price but also because of price dispersion among imported goods invoiced in different currencies. This paper makes the first step in investigating optimal monetary policy under multiple invoicing currencies, leaving the policy implication for the third countries using dominant currencies in future research.

In addition, in the standard two-country New Keynesian framework, it is not surprising that the country having an international reference currency can be worse off in compared to the world of the Mundell-Fleming paradigm (Devereux et al. (2007), Kashiwagi (2017)).[70] This result, however, arises only from a partial role for trade invoicing. Since the dominance of U.S. dollars is prevalent in trade, international security issuance, cross-border banking and international reserve, it is an important future research to investigate optimal policy responses of monetary authorities under a unified environment which incorporates trade, finance and monetary affairs in a dollar standard

[69] From Table 4 in Gopinath (2015), the portion of euro pricers in exporting to the U.S. is significant: 38% for German exporters; 21% for Italian exporters; 18% for French exporters; 16% for Spanish exporters; and 15% for Belgian and Dutch exporters.

[70] Under the non-cooperative Nash equilibrium, Devereux et al. (2007) shows that households in the reference-currency country are worse off in compared to the economy of producer-currency pricing. Consider the two-country world, say, the U.S. and China. If all imported goods in both countries are invoiced in dollars and their prices are sticky in dollars, then dollar prices of imported goods in the U.S. do not respond one-for-one to exchange rate fluctuations while imports in China have complete exchange rate pass-through. Hence, U.S. residents suffer more from inefficient expenditure switching than Chinese residents. By characterizing cooperative monetary policy, Kashiwagi (2017) derives analytical conditions under which the reference-currency country can be better off in compared to the economy of PCP. These two papers assume sticky prices for one period.

(Gopinath and Stein (2018), Gourinchas (2019), Gourinchas et al. (2019)).[71]

Strategic interactions among exporters and their endogenous currency choice are another issue. This paper focuses on equilibrium results under exogenously varying degrees of ERPT and risk sharing. Although we capture the flexible degree of ERPT through the law of large numbers, invoicing currencies are exogenously assigned to exporters and exchange rate movements are passed through to each individual product by either 0% or 100%. This approach is at odd with the finding of Gopinath et al. (2010) who document that there is a large difference in the ERPT of the average good even conditional on a price change. Extending our framework to strategic complementarities in price setting and thereby allowing for endogenous currency choice are in more accordance with an empirical support and we leave it for future research.

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[71] For important contributions from the normative analysis under dominant currency pricing, see Devereux et al. (2007), Goldberg and Tille (2009), Shi and Xu (2010), Egorov and Mukhin (2020) and others.

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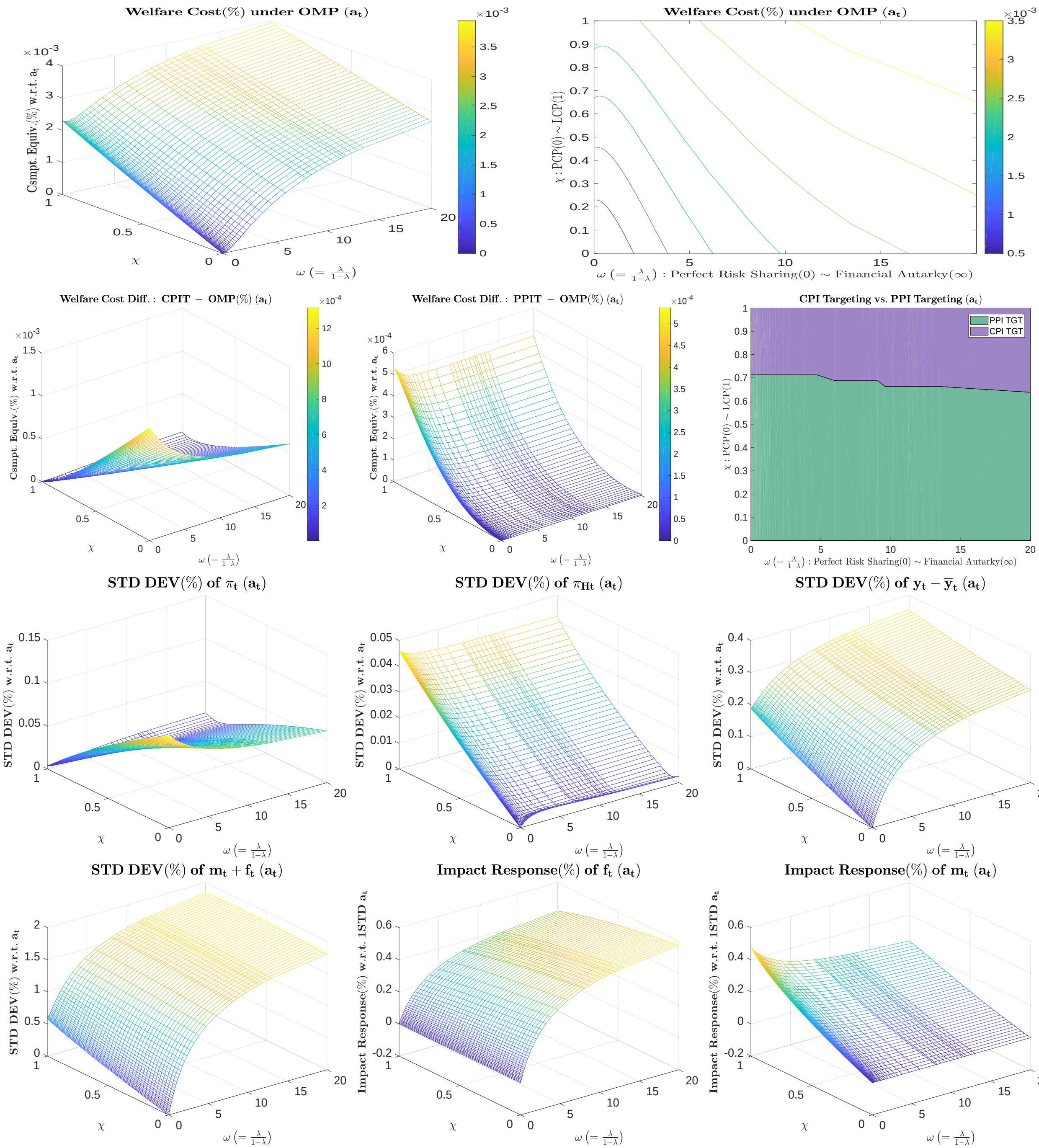
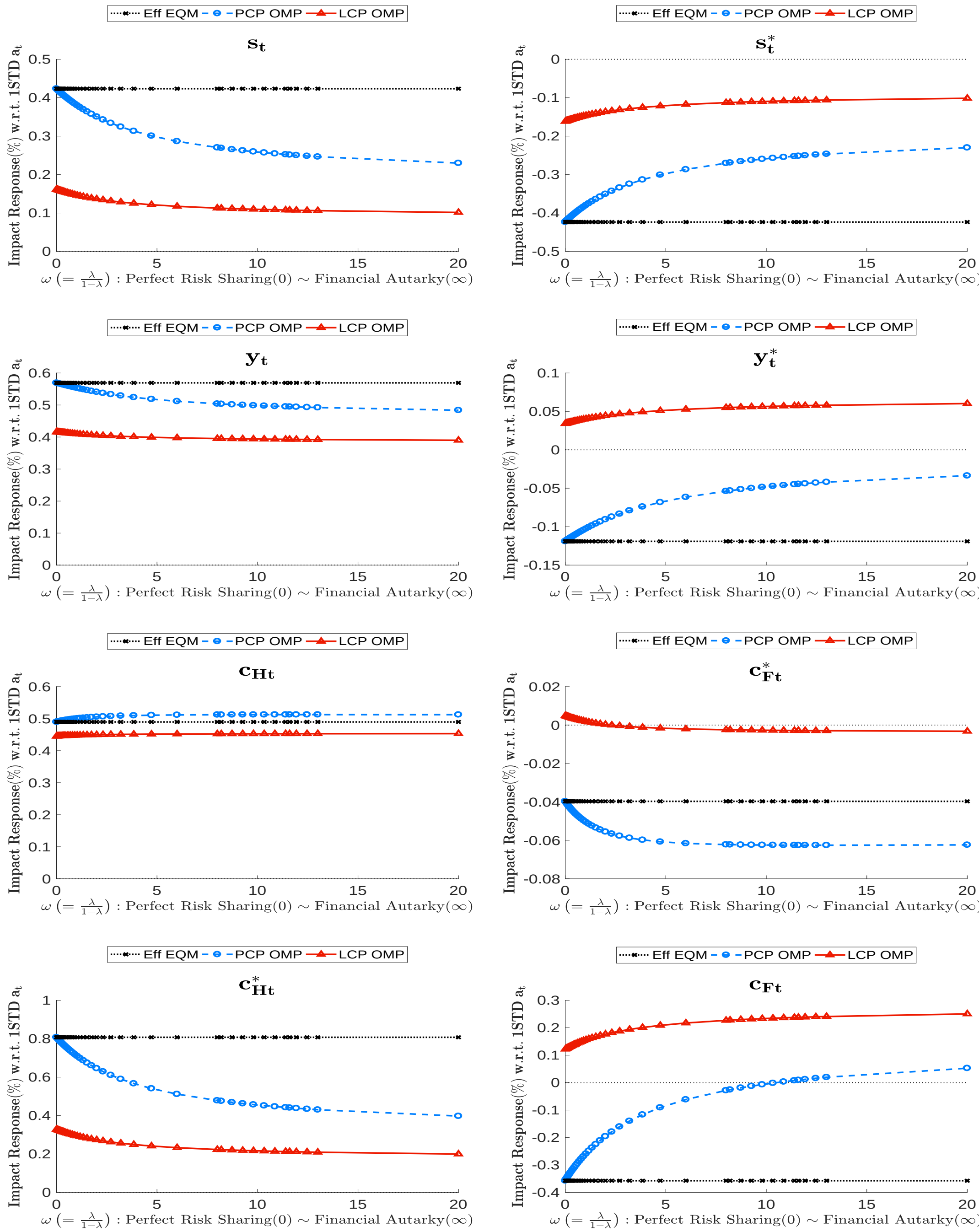
Figure 4: Welfare Cost with respect to a Home Productivity Shock



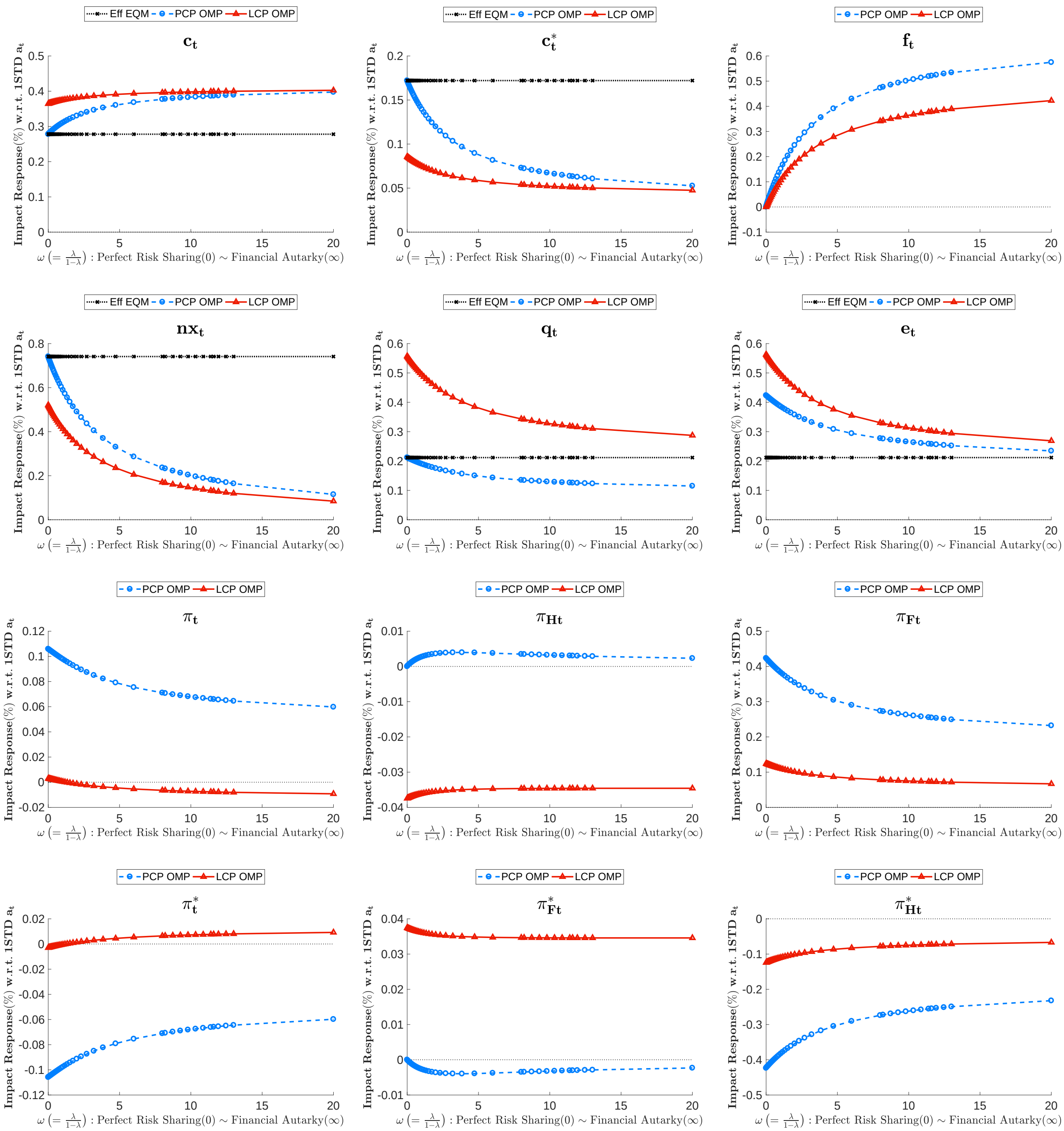

***Note*** – $\chi$ denotes the degree of pricing-to-market from full PCP ($\chi = 0$) to full LCP ($\chi = 1$). $\omega \equiv \frac{\lambda}{1-\lambda}$ represents the degree of financial market integration ranging from perfect risk sharing ($\omega = 0$) to financial autarky ($\omega = \infty$). Welfare costs are measured in consumption equivalent $\eta^C$ (equation (40)). *STD DEV* denotes standard deviation. Impact responses are impulse responses in the first period when shocks occur. The two panels on the top display welfare cost under optimal monetary policy. The first and second figures in the second row shows the difference of welfare costs of CPI and PPI inflation targets from that of optimal policy. The third figure in the second row presents the range of $\chi$ and $\omega$ where CPI inflation targeting incurs lower welfare cost than PPI inflation targeting, represented by a violet area. The green area represents the region where PPI inflation targeting is more effective. $\pi_t$ denotes Home CPI inflation; $\pi_{Ht}$ Home PPI inflation; $y_t - \overline{y}_t$ Home output gap; $f_t$ deviation from perfect risk sharing; $m_t$ deviation from the law of one price.

Figure 5: Impact Responses with respect to a 1 STD Home Productivity Shock



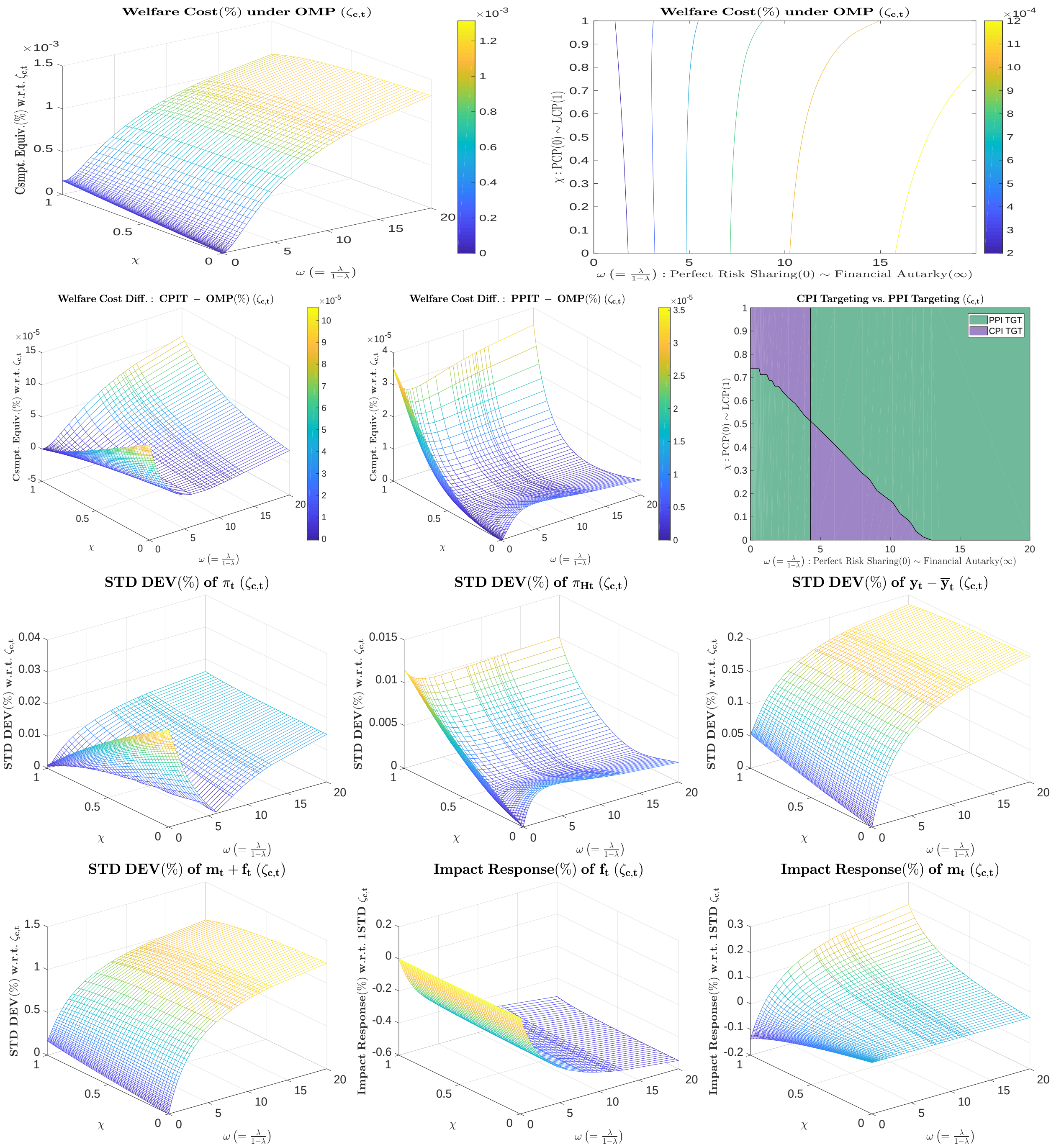

***Note*** – The horizontal axis represents varying degrees of financial market integration, $\omega \equiv \frac{\lambda}{1-\lambda}$, ranging from perfect risk sharing ($\omega = 0$) to financial autarky ($\omega = \infty$). Impact responses are impulse responses in the first period when shocks occur. $Eff\ EQM$ denotes efficient allocations plotted by the bold black dotted line; $PCP\ OMP$ stands for allocations under PCP sticky prices plotted by the blue circled dashed line; $LCP\ OMP$ represents allocations under LCP sticky prices plotted by the red triangular solid line. $s_t$ denotes the price of imported goods in terms of locally-produced goods in Home; $y_t$ Home output; $c_{Ht}$ Home demand for Home-produced goods; $c_{Ft}$ Home demand for Foreign-produced goods; $c_{Ft}^*$ Foreign demand for Foreign-produced goods; $c_{Ht}^*$ Foreign demand for Home-produced goods. Variables with asterisk denote Foreign counterparts.

Figure 6: Impact Responses with respect to a 1 STD Home Productivity Shock



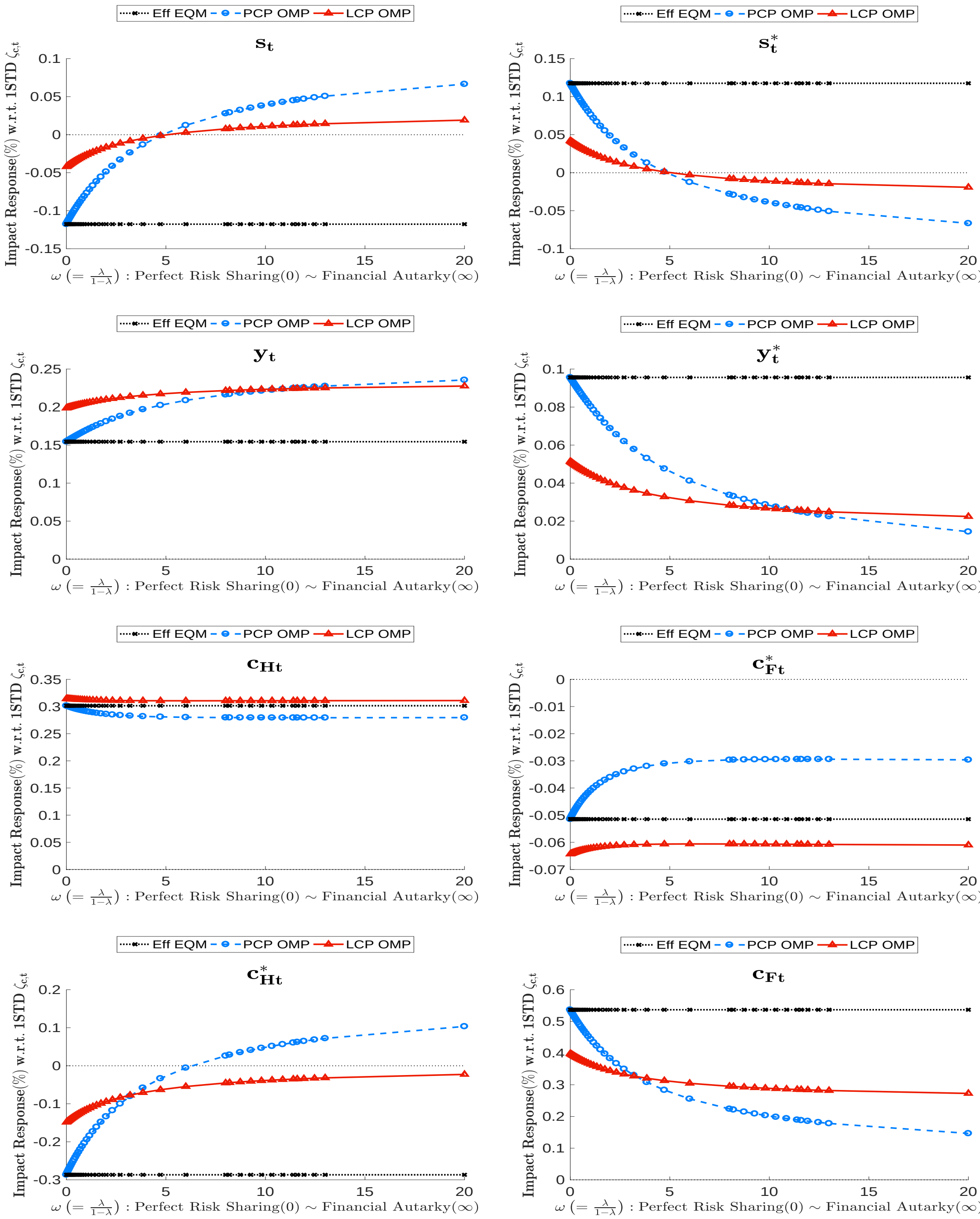

***Note*** – The horizontal axis represents varying degrees of financial market integration, $\omega \equiv \frac{\lambda}{1-\lambda}$, ranging from perfect risk sharing ($\omega = 0$) to financial autarky ($\omega = \infty$). Impact responses are impulse responses in the first period when shocks occur. $Eff\ EQM$ denotes efficient allocations plotted by the bold black dotted line; $PCP\ OMP$ stands for allocations under PCP sticky prices plotted by the blue circled dashed line; $LCP\ OMP$ represents allocations under LCP sticky prices plotted by the red triangular solid line. $c_t$ denotes Home aggregate consumption; $nx_t$ Home net exports; $f_t$ Home demand imbalance; $q_t$ Home real exchange rate; $e_t$ Home nominal exchange rate; $\pi_t$ Home CPI inflation; $\pi_{Ht}$ Home PPI inflation for Home-produced goods; $\pi_{Ft}$ Home PPI inflation for Foreign-produced goods. Variables with asterisk denote Foreign counterparts.

Figure 7: Welfare Cost with respect to a Home Preference Shock



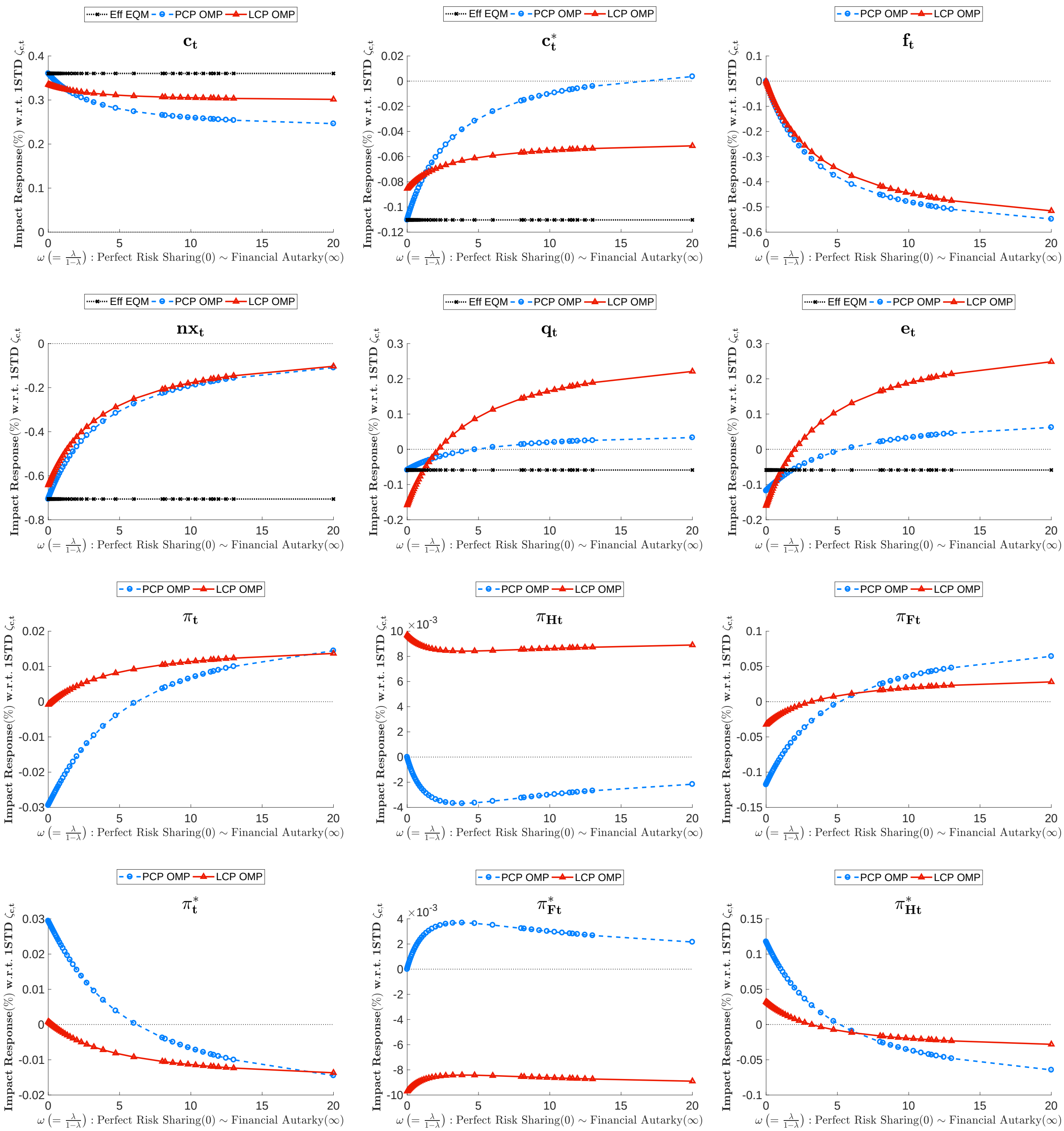

***Note*** – $\chi$ denotes the degree of pricing-to-market from full PCP ($\chi = 0$) to full LCP ($\chi = 1$). $\omega \equiv \frac{\lambda}{1-\lambda}$ represents the degree of financial market integration ranging from perfect risk sharing ($\omega = 0$) to financial autarky ($\omega = \infty$). Welfare costs are measured in consumption equivalent $\eta^C$ (equation (40)). *STD DEV* denotes standard deviation. Impact responses are impulse responses in the first period when shocks occur. The two panels on the top display welfare cost under optimal monetary policy. The first and second figures in the second row shows the difference of welfare costs of CPI and PPI inflation targets from that of optimal policy. The third figure in the second row presents the range of $\chi$ and $\omega$ where CPI inflation targeting incurs lower welfare cost than PPI inflation targeting, represented by a violet area. The green area represents the region where PPI inflation targeting is more effective. $\pi_t$ denotes Home CPI inflation; $\pi_{Ht}$ Home PPI inflation; $y_t - \overline{y}_t$ Home output gap; $f_t$ deviation from perfect risk sharing; $m_t$ deviation from the law of one price.

Figure 8: Impact Responses with respect to a 1 STD Home Preference Shock




***Note*** – The horizontal axis represents varying degrees of financial market integration, $\omega \equiv \frac{\lambda}{1-\lambda}$, ranging from perfect risk sharing ($\omega = 0$) to financial autarky ($\omega = \infty$). Impact responses are impulse responses in the first period when shocks occur. $Eff\ EQM$ denotes efficient allocations plotted by the bold black dotted line; $PCP\ OMP$ stands for allocations under PCP sticky prices plotted by the blue circled dashed line; $LCP\ OMP$ represents allocations under LCP sticky prices plotted by the red triangular solid line. $q_t$ denotes real exchange rate; $s_t$ the price of imported goods in terms of locally-produced goods in Home; $y_t$ Home output; $c_{Ht}$ Home demand for Home-produced goods; $c_{Ft}$ Home demand for Foreign-produced goods; $c^*_{Ft}$ Foreign demand for Foreign-produced goods; $c^*_{Ht}$ Foreign demand for Home-produced goods. Variables with asterisk denote Foreign counterparts.

Figure 9: Impact Responses with respect to a 1 STD Home Preference Shock




***Note*** – The horizontal axis represents varying degrees of financial market integration, $\omega \equiv \frac{\lambda}{1-\lambda}$, ranging from perfect risk sharing ($\omega = 0$) to financial autarky ($\omega = \infty$). Impact responses are impulse responses in the first period when shocks occur. $Eff\ EQM$ denotes efficient allocations plotted by the bold black dotted line; $PCP\ OMP$ stands for allocations under PCP sticky prices plotted by the blue circled dashed line; $LCP\ OMP$ represents allocations under LCP sticky prices plotted by the red triangular solid line. $c_t$ denotes Home aggregate consumption; $nx_t$ Home net exports; $f_t$ Home demand imbalance; $q_t$ Home real exchange rate; $e_t$ Home nominal exchange rate; $\pi_t$ Home CPI inflation; $\pi_{Ht}$ Home PPI inflation for Home-produced goods; $\pi_{Ft}$ Home PPI inflation for Foreign-produced goods. Variables with asterisk denote Foreign counterparts.

## Description of Data used for Pass-Through Regressions

We extend the dataset used for pass-through regressions in Burstein and Gopinath (2014) by updated series. We follow their methodology in constructing trade-weighted measures. Here, we only report source, time range, and concept of our dataset. Other details can be found in the appendix of Burstein and Gopinath (2014). Table 3 reports ERPT for Eurozone and other major developed countries estimated by data from 1985 to 2019. The other subsequent tables show the summary of our dataset for each country. $CPIovr$ denotes log changes in the CPI, which is an expenditure-weighted average of the change in retail prices consumers pay for goods and services, including both domestically produced and imported items. $CPItra$ denotes the log changes in the component of the CPI that is categorized as tradeable. $IPI$, $NER$, and $PPI$ represent log quarterly differences of the import price index, the nominal exchange rate, and the producer price index, respectively.

Table 3: Exchange Rate Pass-Through (1985q1 – 2019q4)

| | CAN | FIN | FRA | DEU | GRC | ITA | JPN | KOR | NLD | ESP | SWE | CHE | GBR | USA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Short-Run | | | | | | | | | | | | | | |
| IPI | 0.74 | 0.22 | 0.17 | 0.36 | 0.10 | 0.53 | 0.76 | 0.82 | 0.52 | 0.75 | 0.43 | 0.17 | 0.40 | 0.17 |
| | (.06) | (.08) | (.09) | (.04) | (.10) | (.05) | (.07) | (.09) | (.06) | (.10) | (.02) | (.05) | (.03) | (.04) |
| CPIT | -.00 | 0.03 | 0.08 | -.02 | 0.01 | 0.07 | -.04 | 0.07 | -.03 | -.04 | 0.06 | 0.07 | 0.03 | 0.08 |
| | (.03) | (.05) | (.10) | (.04) | (.29) | (.03) | (.01) | (.03) | (.13) | (.05) | (.04) | (.04) | (.02) | (.02) |
| Long-Run (2 years) | | | | | | | | | | | | | | |
| IPI | 0.86 | 0.41 | 0.24 | 0.46 | 0.30 | 0.75 | 0.67 | 0.75 | 0.71 | 0.59 | 0.75 | 0.63 | 0.88 | 0.47 |
| | (.12) | (.11) | (.38) | (.07) | (.16) | (.16) | (.12) | (.09) | (.13) | (.24) | (.07) | (.12) | (.08) | (.07) |
| CPIT | 0.07 | 0.41 | 0.24 | 0.01 | 0.18 | 0.08 | 0.07 | 0.18 | 0.43 | 0.11 | 0.30 | 0.24 | 0.14 | 0.18 |
| | (.14) | (.08) | (.17) | (.05) | (.25) | (.14) | (.04) | (.05) | (.16) | (.12) | (.05) | (.12) | (.05) | (.04) |

***Note*** – IPI denotes the import price index and CPIT represents the consumer price index of tradeable items. We redo pass-through regressions in Burstein and Gopinath (2014) by updated series. BIS effective exchange rate (narrow indices) is used for the nominal exchange rate index. Data span 1985q1 to 2019q4 if available: IPI (1985q1-2012q2) and CPIT (1985q1-2019q4) for CAN; IPI (1995q2-2019q4) and CPIT (1985q1-2019q4) for FIN; IPI (1999q2-2019q4) and CPIT (1990q2-2019q4) for FRA; IPI (1985q1-2019q4) and CPIT (1996q2-2019q4) for DEU; IPI (2000q2-2019q4) and CPIT (1999q1-2019q4) for GRC; IPI (1985q1-2019q4) and CPIT (1985q1-2019q4) for ITA; IPI (1985q1-2013q3) and CPIT (1985q1-2019q4) for JPN; IPI (1985q1-2019q4) and CPIT (1985q2-2019q4) for KOR; IPI (2000q2-2019q4) and CPIT (1985q1-2019q4) for NLD; IPI (2005q2-2019q4) and CPIT (1985q1-2019q4) for ESP; IPI (1990q2-2019q4) and CPIT (2000q2-2019q4) for SWE; IPI (2003q4-2019q4) and CPIT (1985q1-2019q4) for CHE; IPI (1985q1-2019q2) and CPIT (1988q2-2019q4) for GBR; IPI (1985q2-2019q4) and CPIT (1985q1-2019q4) for USA.

## Canada (CAN)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 2019q4 | OECD | CPI: All items non-food non-energy Index | 2015=100 |
| CPItra | 1975q2 | 2019q4 | OECD | commodities - inferred using BLS weight | 2015=100 |
| IPI | 1975q2 | 2012q2 | OECD | IMP-End products, inedible | Unit value, local currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

***Note*** – Unit value in Unit implies that the EPI (IPI) series is constructed by the ratio of export (import) values to export (import) volumes evaluated at border prices.

## Finland (FIN)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1996q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1996q2 | 2019q4 | Eurostat | All-items HICP | 2015=100 |
| CPItra | 1975q2 | 2000q1 | OECD | commodities - inferred using BLS weights | 2015=100 |
| CPItra | 2000q2 | 2019q4 | Eurostat | Goods (overall index excluding services) | 2015=100 |
| IPI | 1995q2 | 2019q4 | Eurostat | Manufacturing | 2015=100 |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 1995q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 1995q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

***Note*** – Unit value in Unit implies that the EPI (IPI) series is constructed by the ratio of export (import) values to export (import) volumes evaluated at border prices.

## France (FRA)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1990q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1990q2 | 2019q4 | INSEE | Consumer prices - all items | 2015=100 |
| CPItra | 1990q2 | 2019q4 | INSEE | commodities - inferred using BLS weights | 2015=100 |
| IPI | 1999q2 | 2005q1 | OECD | IMP-Manufactured goods | local currency |
| IPI | 2005q2 | 2019q4 | INSEE | IPI manufacturing | 2015=100, local currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 1995q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 1995q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

## Germany (DEU)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1996q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1996q2 | 2019q4 | Eurostat | All-items HICP | 2015=100 |
| CPItra | 1996q2 | 2019q4 | Eurostat | Goods (overall index excluding services) | 2015=100 |
| IPI | 1975q2 | 2019q4 | DSB | Products of the manufacturing sector | 2015=100, local currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 1995q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 1995q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

***Note*** – DSB in Source represents Destatis Statistisches Bundesamt.

## Greece (GRC)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1996q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1996q2 | 2019q4 | Eurostat | All-items HICP | 2015=100 |
| CPItra | 1989q2 | 1996q1 | OECD | commodities - inferred using BLS weights | 2015=100 |
| CPItra | 1996q2 | 2019q4 | Eurostat | Goods (overall index excluding services) | 2015=100 |
| IPI | 2000q2 | 2019q4 | Eurostat | Manufacturing | 2015=100 |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 1995q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 1995q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

## Italy (ITA)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1996q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1996q2 | 2019q4 | Eurostat | All-items HICP | 2015=100 |
| CPItra | 1975q2 | 2019q4 | OECD | commodities - inferred using BLS weights | 2015=100 |
| IPI | 1975q2 | 1999q1 | OECD | IMP-Manufactured goods, excluding oil products | Unit value, local currency |
| IPI | 1999q2 | 2005q1 | OECD | IMP-Manufactured goods | Unit value, local currency |
| IPI | 2005q2 | 2019q4 | Eurostat | Manufacturing | 2015=100, local currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1981q2 | 1991q1 | IFS | Producer Prices, All Commodities | 2010=100 |
| PPI | 1991q2 | 2000q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 2000q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

***Note*** – Unit value in Unit implies that the EPI (IPI) series is constructed by the ratio of export (import) values to export (import) volumes evaluated at border prices.

## Japan (JPN)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 2019q4 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPItra | 1975q2 | 2019q4 | OECD | commodities - inferred using BLS weights | 2015=100 |
| IPI | 1975q2 | 2013q3 | OECD | IMP-Manufactured goods | Unit value, local currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 2019q4 | OECD | Domestic producer prices - Manufacturing | 2015=100 |

***Note*** – Unit value in Unit implies that the EPI (IPI) series is constructed by the ratio of export (import) values to export (import) volumes evaluated at border prices.

## Republic of Korea (KOR)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1990q1 | IFS | Consumer Prices, All items | 2010=100 |
| CPIovr | 1990q2 | 2019q4 | Bank of Korea | CPI INDEX: ALL ITEMS | 2015=100 |
| CPItra | 1985q2 | 2019q4 | Bank of Korea | CPI INDEX: Commodities | 2015=100 |
| IPI | 1975q2 | 2019q4 | Bank of Korea | Manufacturing products | 2015=100, dollar currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 2019q4 | OECD | Domestic producer prices - Manufacturing | 2015=100 |

## Netherlands (NLD)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1996q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1996q2 | 2019q4 | Eurostat | All-items HICP Index | 2015=100 |
| CPItra | 1975q2 | 2000q1 | OECD | commodities - inferred using BLS weights | 2015=100 |
| CPItra | 2000q2 | 2019q4 | Eurostat | Goods (overall index excluding services) | 2015=100 |
| IPI | 2000q2 | 2019q4 | Eurostat | Manufacturing | 2015=100 |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 1990q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 1990q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

***Note*** – Unit value in Unit implies that the EPI (IPI) series is constructed by the ratio of export (import) values to export (import) volumes evaluated at border prices.

## Spain (ESP)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1976q1 | IFS | Consumer Prices, All items | 2010=100 |
| CPIovr | 1976q2 | 1996q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1996q2 | 2019q4 | Eurostat | All-items HICP | 2015=100 |
| CPItra | 1976q2 | 2001q1 | OECD | commodities - inferred using BLS weights | 2015=100 |
| CPItra | 2001q2 | 2019q4 | Eurostat | Goods (overall index excluding services) | 2015=100 |
| IPI | 2005q2 | 2019q4 | Eurostat | Manufacturing | 2015=100 |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 2000q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 2000q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

## Sweden (SWE)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1996q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1996q2 | 2019q4 | Eurostat | All-items HICP | 2015=100 |
| CPItra | 2000q2 | 2019q4 | Eurostat | Goods (overall index excluding services) | 2015=100 |
| IPI | 1990q2 | 2019q4 | Eurostat | Manufacturing | 2015=100, local currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 1982q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 1982q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

## Switzerland (CHE)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 2005q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 2005q2 | 2019q4 | Eurostat | All-items HICP | 2015=100 |
| CPItra | 1975q2 | 2019q4 | OECD | commodities - inferred using BLS weights | 2015=100 |
| IPI | 2003q4 | 2019q4 | Eurostat | Manufacturing | 2015=100, local currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 2002q3 | IFS | Producer Prices, All Commodities | 2010=100 |
| PPI | 2002q4 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

***Note*** – SFSO in Source represents Swiss Federal Statistical Office. Unit value in Unit implies that the EPI (IPI) series is constructed by the ratio of export (import) values to export (import) volumes evaluated at border prices.

## United Kingdom (GBR)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 1988q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 1988q2 | 2019q4 | ONS | CPI INDEX 00 : ALL ITEMS | 2015=100 |
| CPItra | 1988q2 | 2019q4 | ONS | CPI INDEX: Goods | 2015=100 |
| IPI | 1975q2 | 2011q2 | OECD | IMP-Manufactured goods | Unit value, local currency |
| IPI | 2011q3 | 2019q2 | IFS | Import Prices, All Commodities | 2010=100, local currency |
| NER | 1975q2 | 2019q4 | IFS | period average | National Currency per USD |
| PPI | 1975q2 | 2009q1 | OECD | Domestic producer prices - Manufacturing | 2015=100 |
| PPI | 2009q2 | 2019q4 | OECD | Total producer prices - Manufacturing | 2015=100 |

***Note*** – ONS in Source represents Office for National Statistics. Unit value in Unit implies that the EPI (IPI) series is constructed by the ratio of export (import) values to export (import) volumes evaluated at border prices.

## United States (USA)

| Series | Start | End | Source | Concept | Unit |
|---|---|---|---|---|---|
| CPIovr | 1975q2 | 2002q1 | OECD | CPI: All items non-food non-energy | 2015=100 |
| CPIovr | 2002q2 | 2019q4 | Eurostat | All-items HICP | 2015=100 |
| CPItra | 1975q2 | 2019q4 | BLS | CPI for All Urban Consumers: Commodities in U.S. City Average, CUUR0000SAC | 1982-1984=100 |
| IPI | 1985q2 | 2019q4 | BLS | Import Price Index (End Use): All imports excluding petroleum, IREXPET | 2000=100 |
| PPI | 1975q2 | 1986q1 | OECD | Total producer prices - Manufacturing | 2015=100 |
| PPI | 1986q2 | 2019q4 | BLS | Producer Price Index by Industry: Total Manufacturing Industries, PCUOMFGOMFG | 1984M12=100 |

## Quantitative Importance of Internal and External Targets in the Loss Function

In section 4.1, we present the analytical representation of the loss function. Recall the quadratic loss function given by

$$L_t = \left( \begin{array}{ll} + & \left(\frac{\sigma}{D}+\phi\right)\left(\widetilde{y}_t^R\right)^2 + (\sigma+\phi)\left(\widetilde{y}_t^W\right)^2 + \frac{\epsilon\nu(2-\nu)}{4D}\left(m_t+f_t\right)^2 \\ + & \left(\frac{\xi}{2}\right)\left[\left(\frac{\nu}{2}\right)\sigma_{P_H,t}{}^2 + \left(1-\frac{\nu}{2}\right)\sigma_{P_H^*,t}{}^2 + \left(\frac{\nu}{2}\right)\sigma_{P_F^*,t}{}^2 + \left(1-\frac{\nu}{2}\right)\sigma_{P_F,t}{}^2\right] \end{array} \right), \tag{41}$$

where $\sigma^2_{P_H,t}$, $\sigma^2_{P_H^*,t}$, $\sigma^2_{P_F^*,t}$ and $\sigma^2_{P_F,t}$ are measures of price dispersion across firms. Note that the welfare loss boils down to zero under efficient allocations. The life-time global welfare loss, $\mathfrak{L}_t \equiv \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} L_\tau = \mathfrak{L}_t^y + \mathfrak{L}_t^{mf} + \mathfrak{L}_t^p$, can be decomposed into three parts:

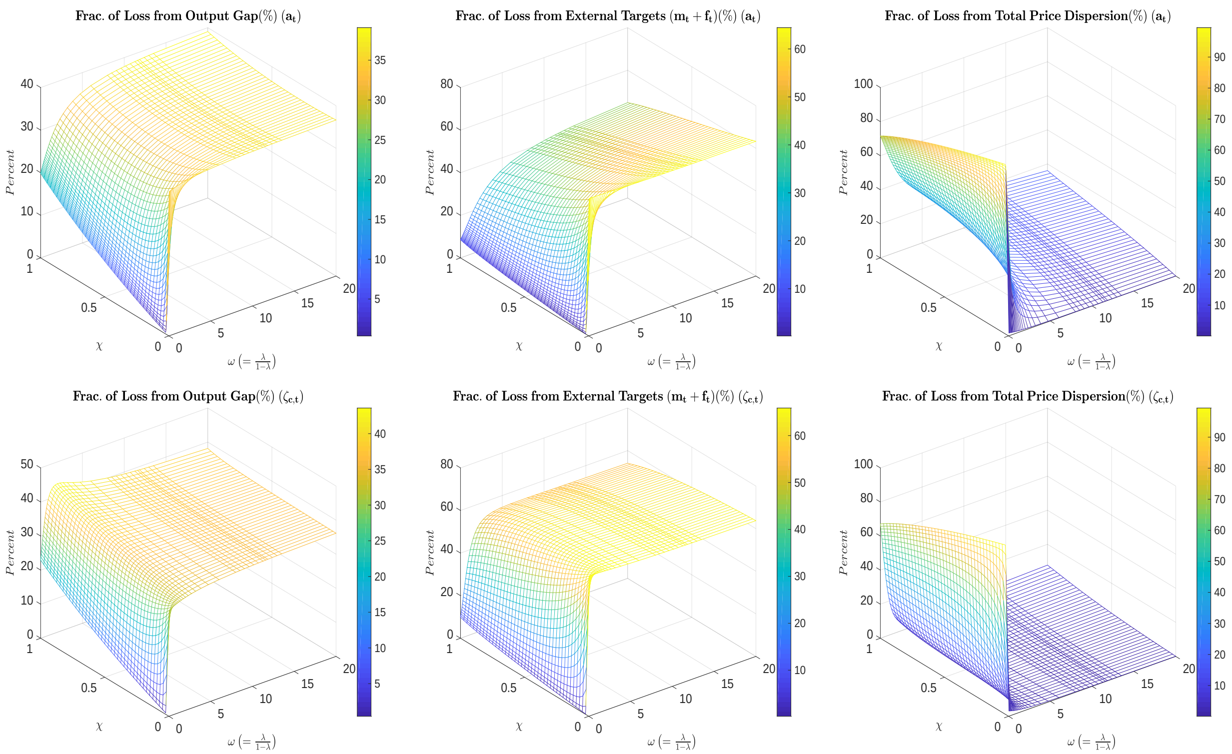
- loss from output gap, $\mathfrak{L}_t^y = \left(\frac{\sigma}{D}+\phi\right)\left(\widetilde{y}_t^R\right)^2 + (\sigma+\phi)\left(\widetilde{y}_t^W\right)^2 + \beta\mathbb{E}_t\mathfrak{L}_{t+1}^y$;
- loss from external targets (currency misalignment plus demand imbalance), $\mathfrak{L}_t^{mf} = \frac{\epsilon\nu(2-\nu)}{4D}\left(m_t+f_t\right)^2 + \beta\mathbb{E}_t\mathfrak{L}_{t+1}^{mf}$;
- loss from price dispersion, $\mathfrak{L}_t^p = \left(\frac{\xi}{2}\right)\left[\left(\frac{\nu}{2}\right)\sigma_{P_H,t}{}^2 + \left(1-\frac{\nu}{2}\right)\sigma_{P_H^*,t}{}^2 + \left(\frac{\nu}{2}\right)\sigma_{P_F^*,t}{}^2 + \left(1-\frac{\nu}{2}\right)\sigma_{P_F,t}{}^2\right] + \beta\mathbb{E}_t\mathfrak{L}_{t+1}^p$.

We plot $\mathfrak{L}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} L_\tau$ in terms of consumption-equivalent measure in the first chart of figures 4 and 7 with respect to a shock to productivity and preference in Home. To be more specific, Figure 10 displays the relative importance among output gap, price dispersion, and external targets in the loss function under varying degrees of exchange rate pass-through ($\chi$) and cross-country risk sharing ($\omega$): charts in the first row show shares in the loss with respect to Home productivity shocks and charts in the second row present those with respect to Home preference shocks.

It can be clearly seen that price dispersion incurs much larger welfare loss than output gap and external targets if the global economy is close to perfect financial integration. Under perfect financial markets, price dispersion accounts for more than 70% of total welfare loss regardless of the degree of exchange rate pass-through ($\chi$). On the other hand, if the degree of cross-country risk sharing is low ($\omega > 5$ or $\lambda > 0.83$), price dispersion is not the main concern. Under the low degree of risk sharing, output gap and external targets account for more than 80% of total welfare loss. Output gap tends to have a greater portion of welfare loss in the lower exchange rate pass-through ($\chi \to 1$) while external targets do so in the higher exchange rate pass-through ($\chi \to 0$).

In Figure 11, we further decompose price dispersion into two terms and compare their shares in loss.

Figure 10: Shares of Policy Targets in Welfare Loss



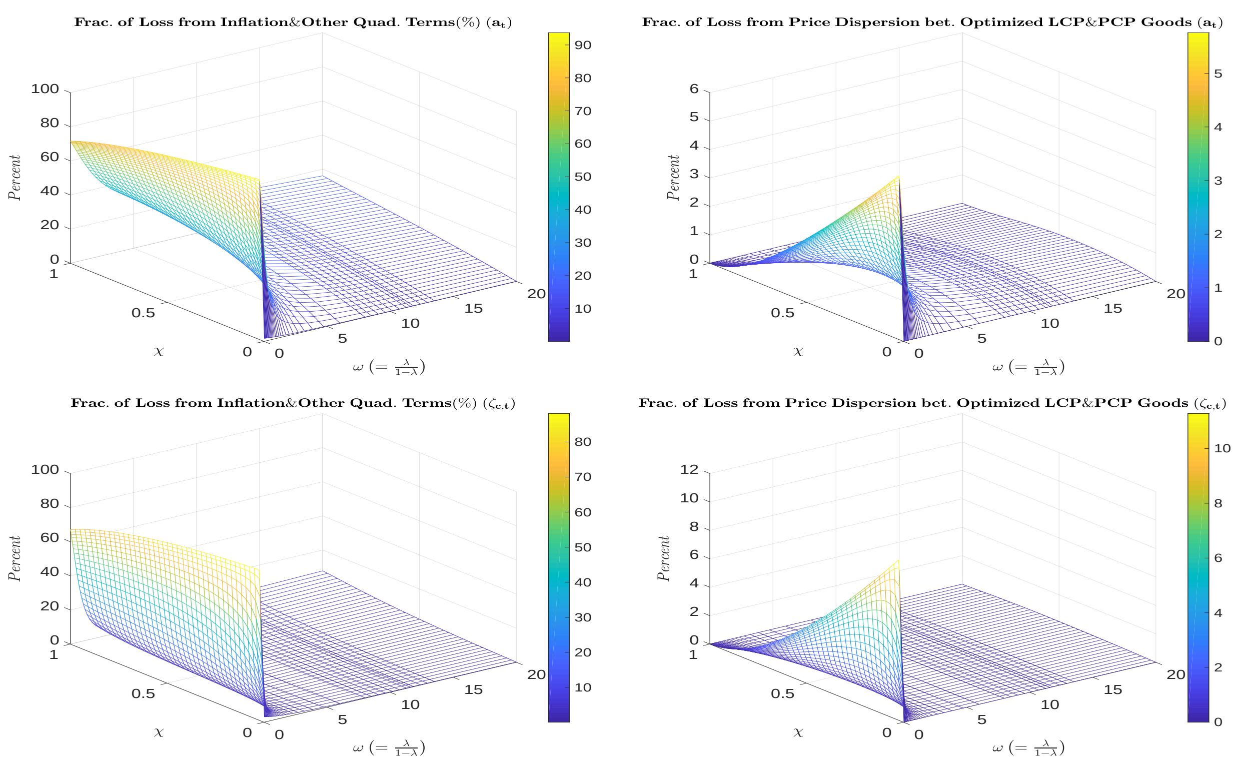

***Note*** − Three charts in the first row show shares of policy targets in global welfare loss under Home productivity shocks ($a_t$) and three charts in the second row show those under Home preference shocks ($\zeta_{c,t}$).

From equations (27) and (32), price dispersion can be rewritten out as

$$
\begin{aligned}
&\left(\frac{\xi}{2}\right)\left[\left(\frac{\nu}{2}\right)\sigma_{P_H,t}{}^2+\left(1-\frac{\nu}{2}\right)\sigma_{P_H^*,t}{}^2+\left(\frac{\nu}{2}\right)\sigma_{P_F^*,t}{}^2+\left(1-\frac{\nu}{2}\right)\sigma_{P_F,t}{}^2\right]\\
&=\left(\begin{array}{l}
+\left(\frac{\xi}{2\delta}\right)\left(1-\frac{\nu}{2}\right)\frac{\chi(1-\chi)^2(1-\theta)^2}{\theta}\left(\left[\pi_{Ht}^{L,o*}-\pi_{Ht}^{P,o}+m_t\right]^2+\left[\pi_{Ft}^{L,o}-\pi_{Ft}^{P,o*}-m_t\right]^2\right)\\
+\left(\frac{\xi}{2\delta}\right)\left[\left(\frac{\nu}{2}\right)(\pi_{H,t})^2+\left(1-\frac{\nu}{2}\right)\left(\pi_{H,t}^*+\Delta e_t\right)^2+\left(\frac{\nu}{2}\right)\left(\pi_{F,t}^*\right)^2+\left(1-\frac{\nu}{2}\right)(\pi_{F,t}-\Delta e_t)^2\right]\\
+\left(\frac{\xi}{2\delta}\right)\left(1-\frac{\nu}{2}\right)\left(\begin{array}{l}
+\left(\pi_{H,t}^*-\pi_{F,t}\right)^2-\left(\pi_{H,t}^*-\pi_{F,t}+\Delta e_t\right)^2-(\Delta e_t)^2\\
+(1-\chi)\left[1+2\theta\chi(1-\chi)\right](e_t-e_{t-1})^2+\frac{2(1-\chi)}{\theta}(m_t-\theta m_{t-1})^2\\
+(1-\chi)\left(\begin{array}{l}
\chi(1-\chi)(1-\theta)\left(\pi_{Ht}^{L,o*}-\pi_{Ht}^{P,o}-\pi_{Ft}^{L,o}+\pi_{Ft}^{P,o*}+2m_t\right)\\
+\pi_{H,t}^{P}-\pi_{H,t-1}^{P}-\pi_{F,t}^{P*}+\pi_{F,t-1}^{P*}+\pi_{H,t}^*-\pi_{F,t}-2m_t+2m_{t-1}+e_t-e_{t-1}
\end{array}\right)^2\\
-(1-\chi)\left(\begin{array}{l}
\chi(1-\chi)(1-\theta)\left(\pi_{Ht}^{L,o*}-\pi_{Ht}^{P,o}-\pi_{Ft}^{L,o}+\pi_{Ft}^{P,o*}+2m_t\right)\\
+\pi_{H,t}^{P}-\pi_{H,t-1}^{P}-\pi_{F,t}^{P*}+\pi_{F,t-1}^{P*}+\pi_{H,t}^*-\pi_{F,t}-2m_t+2m_{t-1}
\end{array}\right)^2
\end{array}\right)
\end{array}\right).
\end{aligned}
\tag{42}
$$

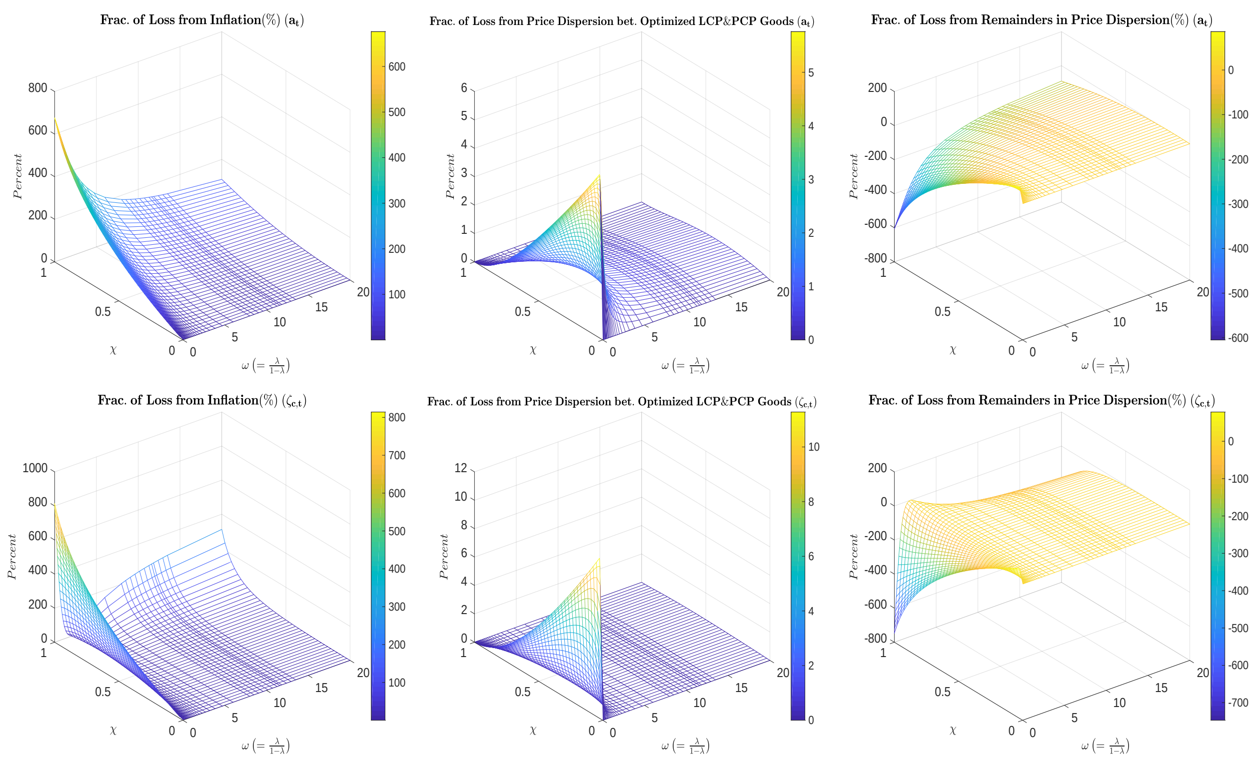
Due to algebraic complexity, equation (42) does not allow for decomposition between inflation and consumer price dispersion among PCP and LCP goods and we cannot separate their exact shares in welfare loss. Instead, we can only distinguish the price dispersion between PCP and LCP goods which are newly priced in the current period from total price dispersion. It turns out its share ranges from 0% to 5% with respect to productivity shocks and from 0% to 10% with respect to preference shocks. The share tends to get larger as the financial market converges perfect risk sharing under incomplete exchange rate pass-through where $\chi$ ranges from 0 to 0.5. We cannot decompose price dispersion terms further because of the presence of negative quadratic terms. Due to negative quadratic terms, the other inflation terms can take more than 100% in the share of welfare loss, which does not convey any economic meaning (see Figure 12).

Figure 11: Shares of Price Dispersion Terms in Welfare Loss




***Note*** $-$ Two charts in the first row show shares of price dispersion terms in global welfare loss under Home productivity shocks ($a_t$) and two charts in the second row show those under Home preference shocks ($\zeta_{c,t}$).

Figure 12: Shares of Price Dispersion Terms in Welfare Loss




***Note*** $-$ Three charts in the first row show shares of price dispersion terms in global welfare loss under Home productivity shocks $(a_t)$ and three charts in the second row show those under Home preference shocks $(\zeta_{c,t})$.